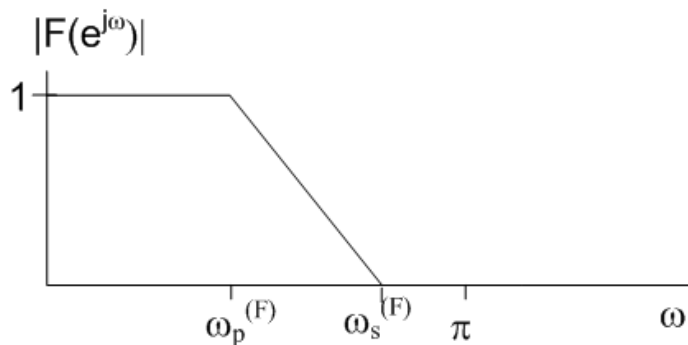


Write in each answer paper your name, department, student number, the course name and code, and the date. Number each paper you submit and denote the total no. of pages. 4 problems, 32 points total. Exam problems in English only. Please feel free to answer in Finnish or English. No additional material is allowed in the exam.

1. (8p) Explain *briefly* the following concepts:
 - (a) Noble identities
 - (b) Input-output relations of interpolator in time and frequency domains
 - (c) Nyquist filter
 - (d) Farrow filter
2. (8p) Consider the frequency-response masking approach:

$$H_{FM}(z) = H(z)I_1(z) + G(z)I_2(z)$$

where $I_1(z)$ and $I_2(z)$ are interpolation filters, $G(z)$ is the complementary filter of $H(z)$. $H(z)$ in turn is obtained from shaping filter $F(z)$ using the upsampling factor (sparsity factor) $L = 3$. The magnitude response of $F(z)$ is given below



- (a) Sketch $H(z)$ and $G(z)$.
- (b) Sketch the 9 different single-band filters (i.e. no multiple pass-bands and excluding the trivial all-pass and all-stop filters) $H_{FM}(z)$ that can be implemented using the frequency-masking approach.

3. (8p) Consider the product filter given by

$$P(z) = az^2 + bz + c + dz^{-1} + ez^{-2}$$

- (a) Determine a, b, c, d, e so that the resulting filter is a perfect reconstructing one.
- (b) Factorize $P(z)$ to analysis and synthesis filters so that the resulting filter bank is orthogonal.
4. (8p) Design a three-channel perfect reconstruction QMF filter bank whose analysis filters are given by

$$\begin{aligned}H_0(z) &= 1 + z^{-1} + 2z^{-2} \\H_1(z) &= 2 + 4z^{-1} + z^{-2} \\H_2(z) &= 1 + 2z^{-1} + z^{-2}\end{aligned}$$

and draw a block diagram of a computationally efficient realization of the filter bank.