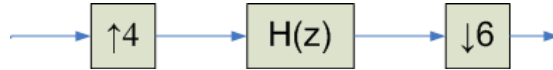


Write in each answer paper your name, department, student number, the course name and code, and the date. Number each paper you submit and denote the total no. of pages. 4 problems, 32 points total. Exam problems in English only. Please feel free to answer in Finnish or English. No additional material is allowed in the exam.

1. (8p) Explain *briefly* the following concepts:

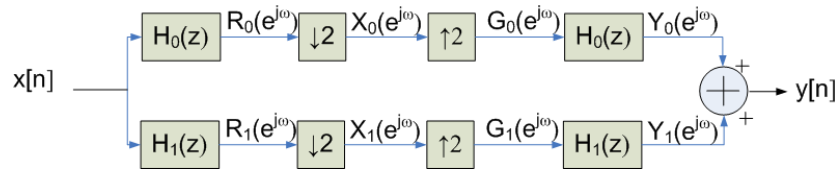
- (a) Noble identities
- (b) Input-output relations of decimator in time and frequency domains
- (c) Uniform DFT filter bank
- (d) Discrete wavelet transform

2. (8p) Consider the sampling rate converter below:



Explain step-by-step by drawing block diagrams how this system can be transformed into a computationally efficient sampling rate converter.

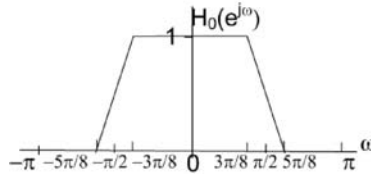
3. (8p) Consider the analysis/synthesis system shown below.



The low-pass filter $H_0(z)$ and high-pass filter $H_1(z)$ in the analyzer and synthesizer are identical, and their Fourier-transforms are related as

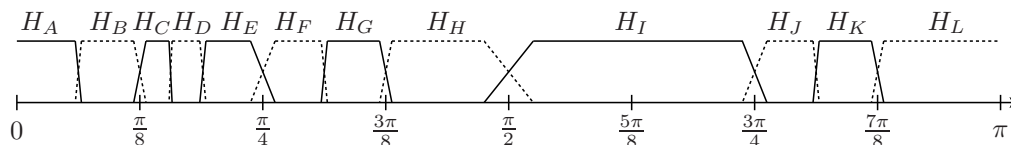
$$H_1(e^{j\omega}) = H_0(e^{j(\omega-\pi)}),$$

and $H_0(e^{j\omega})$ is depicted below.

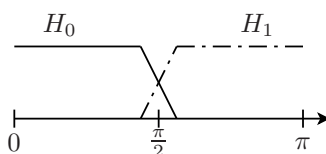


- Draw the distortion transfer function and the alias term.
- Is the system alias-free, perfect-reconstruction, or both? If not, how would you modify the system block diagram to make the system alias-free and perfect-reconstruction?

4. (8p) Consider a 12-channel single-level analysis filter bank with unequal passband widths that are shown below.



- Assuming the parent lowpass and highpass filters $H_0(z)$ and $H_1(z)$ with the spectra shown below, sketch $H_0(z^2)$, $H_0(z^4)$, $H_0(z^8)$, $H_1(z^2)$, $H_1(z^4)$, and $H_1(z^8)$.



- Draw the block diagram of the equivalent four-level analysis filter bank. The filter bank should exploit only down-sampling by two, and the parent filters $H_0(z)$, $H_1(z)$.
- Express the filters $H_A(z)$, $H_B(z)$, $\dots$, $H_L(z)$ in terms of the parent filters $H_0(z)$ and $H_1(z)$.