

Write in each answer paper your name, department, student number, the course name and code, and the date. Number each paper you submit and denote the total no. of pages. 5 problems, 30 points total. Papers in English only. The BETA mathematical tables can be utilized – you can borrow a copy from the exam supervisor if you do not have your own. A basic calculator can be used (no memory, no graphics).

The homework bonus will be valid for possible future exams too.

1. (1p each) Define and describe briefly (2..3 lines of text) the following concepts:

- a) Echo cancellation
- b) Viterbi algorithm
- c) DFE
- d) Water pouring theorem
- e) OFDM
- f) ZF equalizer

2. Matched filter (6p):

- a) (3p) Define the matched filter (MF) concept in an AWGN channel. Give the solution in both time-domain and frequency-domain forms.
- b) (3p) The Schwarz inequality can be expressed in the form

$$\left| \int_{-\infty}^{\infty} H_T(f) H_R(f) df \right|^2 \leq \int_{-\infty}^{\infty} |H_T(f)|^2 df \int_{-\infty}^{\infty} |H_R(f)|^2 df \quad (1)$$

where the transfer functions with subscripts T and R refer to transmit and receive filters. Show that the equality holds (i.e., the left side equals to the right side) for your MF solution.

3. Adaptive filters (6p):

Let us consider a discrete-time model for a communication system in a linear channel (sampled at the symbol rate). The received signal samples $r(k)$ are filtered by an N -tap FIR filter (equalizer). The equalizer output $y(k)$ can be expressed as

$$y(k) = \mathbf{h}_r^T \mathbf{r}(k) \quad (2)$$

where \mathbf{h}_r and \mathbf{r} are N -dimensional column vectors. Draw the receiver block diagram and derive the MSE gradient (MSEG) adaptive algorithm to update the equalizer coefficients. What is the optimal equalizer solution and in what conditions does the adaptive algorithm reach it? Discuss the convergence issues too.