4. Nyquist and Matched filtering criteria (8p bonus):

We want to design a Root-Nyquist transmit and receive filter pair which are matched to each other and also meet the Nyquist criterion (no ISI).

a) (3p) The ideal Nyquist spectrum is *piecewise constant* with the rolloff factor $\alpha = 0.5$. Express the spectrum of the transmit filter with a formula (as simple as possible!). Draw a plot of the spectrum magnitude.

b) (1p) Show that the above spectrum is a Nyquist spectrum (e.g., graphically).

c) (4p) Solve for the (ideal) impulse response as well using the inverse Fourier transform.

5. Channel capacity (6p):

Consider the transmission of signal $x(t)$ over a linear channel with associated impulse response $c(t)$ and frequency response $C(f)$. The output waveform of the channel is then $r(t) = c(t)*x(t)$, where '*' denotes convolution. The output of the channel is thereafter corrupted by colored noise $n(t)$ with power spectral density (PSD) $S_n(f)$.

a) (4p) Solve for the optimum transmit power spectrum $S_x(f)$ that maximizes the channel capacity given a water-filling level $L = 4$ (see Equation (3) below). To help you solve the problem, Figure 1 below provides you with the PSD of the noise $S_n(f)$ and the magnitude squared of the channel transfer function. Assume that $S_n(f) = \infty$, for $|f| > 4$ Hz.

b) (2p) Determine the resulting transmit power P_x and channel capacity given the optimized transmit power spectrum in a).

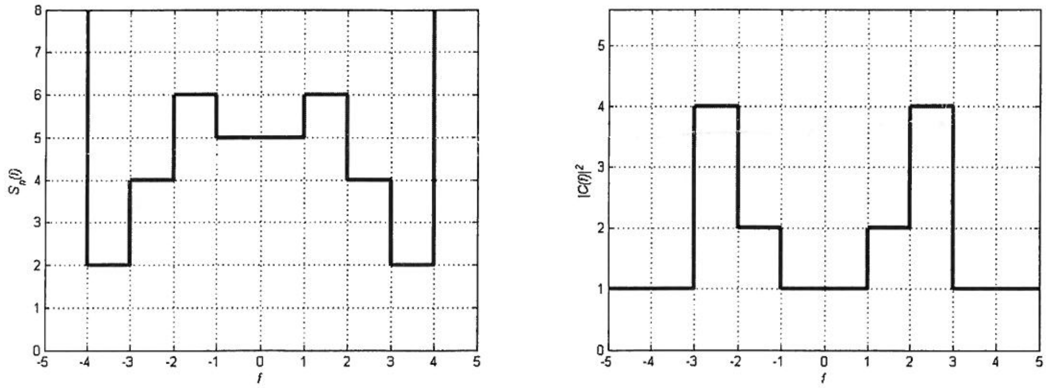

Figure 1: PSD of the noise $S_n(f)$ in W/Hz and channel gain $|C(f)|^2$ versus frequency f in Hz.

Hint: The optimal power spectrum is obtained with the water-pouring theorem as:

$$S_{x,opt}(f) = L - S_n(f)/|C(f)|^2 \tag{3}$$

whenever resulting $S_{x,\mathrm{opt}}(f)$ is positive (zero otherwise) and the water-filling L is determined so that the total transmit power is limited, i.e.,

$$P_x = \int_{-\infty}^{\infty} S_{x,opt}(f)df \tag{4}$$