Aalto University, School of Electrical Engineering **Exam Jan 12 2011**
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S-88.2111 Signal Processing in Telecommunications I (3 cp)

Write in each answer paper your name, department, student number, the course name and code, and the date. Number each paper you submit and denote the total no. of pages. 5 problems, 30 points total. Papers in English only. The BETA mathematical tables can be utilized – you can borrow a copy from the exam supervisor if you do not have your own. A basic calculator can be used (no memory, no graphics).

The homework bonus will be valid for possible future exams too.

1. (1p each) Define and describe *briefly* (2..3 lines of text) the following concepts:

- **a)** Echo cancellation
- **b)** Viterbi algorithm
- **c)** DFE
- **d)** Water pouring theorem
- **e)** OFDM
- **f)** ZF equalizer

2. Matched filter (6p):

a) (3p) Define the matched filter (MF) concept in an AWGN channel. Give the solution in both time-domain and frequency-domain forms.

b) (3p) The Schwarz inequality can be expressed in the form

$$\left|\int_{-\infty}^{\infty} H_T(f) H_R(f) df\right|^2 \leq \int_{-\infty}^{\infty} |H_T(f)|^2 df \int_{-\infty}^{\infty} |H_R(f)|^2 df \qquad (1)$$

where the transfer functions with subscripts T and R refer to transmit and receive filters. Show that the equality holds (i.e., the left side equals to the right side) for your MF solution.

3. Adaptive filters (6p):

Let us consider a discrete-time model for a communication system in a linear channel (sampled at the symbol rate). The received signal samples $r(k)$ are filtered by an N-tap FIR filter (equalizer). The equalizer output $y(k)$ can be expressed as

$$y(k) = \mathbf{h}_R^T \mathbf{r}(k) \qquad (2)$$

where $\mathbf{h}_R$ and $\mathbf{r}$ are N-dimensional column vectors. Draw the receiver block diagram and derive the MSE gradient (MSEG) adaptive algorithm to update the equalizer coefficients. What is the optimal equalizer solution and in what conditions does the adaptive algorithm reach it? Discuss the convergence issues too.

4. Nyquist and Matched filtering criteria (8p bonus):

We want to design a Root-Nyquist transmit and receive filter pair which are matched to each other and also meet the Nyquist criterion (no ISI).

a) (3p) The ideal Nyquist spectrum is *piecewise constant* with the rolloff factor $\alpha = 0.5$. Express the spectrum of the transmit filter with a formula (as simple as possible!). Draw a plot of the spectrum magnitude.

b) (1p) Show that the above spectrum is a Nyquist spectrum (e.g., graphically).

c) (4p) Solve for the (ideal) impulse response as well using the inverse Fourier transform.

5. Channel capacity (6p):

Consider the transmission of signal $x(t)$ over a linear channel with associated impulse response $c(t)$ and frequency response $C(f)$. The output waveform of the channel is then $r(t) = c(t)*x(t)$, where '*' denotes convolution. The output of the channel is thereafter corrupted by colored noise $n(t)$ with power spectral density (PSD) $S_n(f)$.

a) (4p) Solve for the optimum transmit power spectrum $S_x(f)$ that maximizes the channel capacity given a water-filling level $L = 4$ (see Equation (3) below). To help you solve the problem, Figure 1 below provides you with the PSD of the noise $S_n(f)$ and the magnitude squared of the channel transfer function. Assume that $S_n(f) = \infty$, for $|f| > 4$ Hz.

b) (2p) Determine the resulting transmit power P_x and channel capacity given the optimized transmit power spectrum in a).

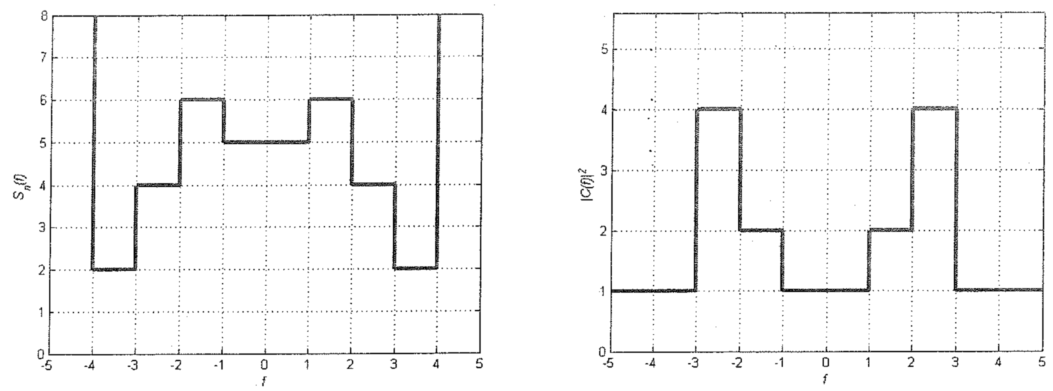

Figure 1: PSD of the noise $S_n(f)$ in W/Hz and channel gain $|C(f)|^2$ versus frequency f in Hz.

Hint: The optimal power spectrum is obtained with the water-pouring theorem as:

$$S_{x,opt}(f) = L - S_n(f)/|C(f)|^2 \tag{3}$$

whenever resulting $S_{x,\text{opt}}(f)$ is positive (zero otherwise) and the water-filling L is determined so that the total transmit power is limited, i.e.,

$$P_x = \int_{-\infty}^{\infty} S_{x,opt}(f)df \tag{4}$$