Aalto University School of Science and Technology
Department of Communications and Networking
S-38.3141 Teletraffic Theory, Spring 2010

Examination
13.1.2011
Aalto

1. Consider the following circuit-switched access network. There are two access links ($l = 1, 2$) and a joint trunk network link $l = 3$. Let C_l denote the capacity of link l. There are two traffic classes $s = 1, 2$ with class s using links s and 3. Traffic consists of ordinary telephone calls that are IID and exponentially distributed with mean $1/\mu$. New calls of class s arrive according to an independent Poisson process with intensity λ_s. Let B_s denote the end-to-end blocking probability for class s. Assume now that $C_1 = C_2 = 1$, $C_3 = 2$. Determine B_s for all s using the following three methods:

   (a) Determine the exact value for B_1 (as a function of $a_1 = \lambda_1/\mu$).

   (b) By applying the Reduced Load Approximation method, give a system of equations from which an approximative value for B_1 can be solved (as a function of $a_1 = \lambda_1/\mu$ and $a_2 = \lambda_2/\mu$).

2. Consider an open queueing network with two nodes. New customers arrive with intensity $\lambda = 1/5$ customers/min. All customers enter first node 1. The service times in node 1 are independent and exponentially distributed with mean 1 min. Upon departure from node 1, the customer returns to node 1 with probability 1/2 and enters node 2 with probability 1/2. The service times in node 2 are independent and exponentially distributed with mean 2 min. Upon departure from node 2, the customer returns to node 2 with probability 1/2 and exits the whole network with probability 1/2.

   (a) Let λ_i denote the average customer flow through node i. Determine λ_1 and λ_2.

   (b) Let N denote the (steady-state) number of customers in the whole network. Determine $P\{N = 0\}$.

   (c) Let T denote the total time that a customer spends in the network. Determine $E[T]$.

3. Let $0 < p < 1$. Derive the rate function $I(x)$, $p < x < 1$, for a random variable obeying the Bernoulli(p) distribution.

4. Consider a linear network with two links $j = 1, 2$. The long route $r = 0$ uses both links, while the short routes $r = 1, 2$ use a single link (r). Let $n_r > 0$ denote the number of flows on route r. Both links have capacity 1. For any positive $\alpha \neq 1$, derive the proportional fair bandwidth shares $\mathbf{x} = (x_0, x_1, x_2)$ for the flows on different routes, which maximize the total utility

$$n_0 \log x_0 + n_1 \log x_1 + n_2 \log x_2.$$

5. Consider the P2P file sharing application. Assume that the system is download-constrained. Based on the Qiu-Srikant fluid model

$$\begin{aligned} x'(t) &= \lambda - \min\{cx(t), \mu(\eta x(t) + y(t))\}, \\ y'(t) &= \min\{cx(t), \mu(\eta x(t) + y(t))\} - \gamma y(t), \end{aligned}$$

solve the steady-state mean number of leechers ($\bar{x}$) and non-permanent seeds ($\bar{y}$).