AALTO UNIVERSITY
Department of Communications and Networking
S-38.3143 Queueing Theory, Fall 2010

Examination
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Aalto

1. Consider a single-server queue. The system is empty at time 0. New customers arrive at times 1, 2, and 4. Their service times are 6, 3, and 3, respectively. For each of the three service disciplines given below, determine the departure times of all three customers:

   (a) FIFO, (b) PS, (c) SRPT.

2. Consider a renewal sequence (T_n) with finite mean $E[T] < \infty$. Let $T^e(t)$ denote the corresponding elapsed lifetime process. Utilizing the theory of regenerative processes, determine the Laplace transform $E[e^{-sT^e}]$ of the steady-state elapsed lifetime distribution (as a function of the mean value $E[T]$ and the Laplace transform $E[e^{-sT}]$).

3. Consider an M/G/1 queue with $\rho \geq 1$. Show that the unfinished work process $U(t)$ satisfies the following inequality for any $x \geq 0$:

$$\lim_{h \to 0+} \frac{1}{h} E[U(t+h) - U(t) \mid U(t) = x] \geq 0.$$

4. Consider an M/E_2/1-FIFO queue with $\rho < 1$. So we assume that the service times follow the Erlang distribution with two phases,

$$P\{S \leq x\} = 1 - e^{-2\mu x}(1 + 2\mu x).$$

   Let $X(t)$ denote the queue length at time t. In addition, let $Z(t)$ denote the phase of the customer in service (if any). If the system is empty, then $Z(t) = 0$. The pair $(X(t), Z(t))$ is a Markov process with state space

$$\mathcal{S} = (0,0) \cup \{(n,z) : n \in \{1,2,\ldots\}, z \in \{1,2\}\}.$$

   (a) Draw the state transition diagram of the Markov process $(X(t), Z(t))$.
   (b) Utilizing the Pollaczek-Khinchin mean value formulas, determine the mean steady-state queue length $E[X]$ (as a function of λ and μ).

5. Utilizing the properties of the LIFO-PR discipline, prove that the Laplace transform of the steady-state busy period distribution in a stable M/G/1 queue satisfies the Kendall-Takács functional equation:

$$B^*(s) = S^*(s + \lambda - \lambda B^*(s)),$$

   where

$$B^*(s) = E[e^{-sB}], \qquad S^*(s) = E[e^{-sS}].$$