I. The multipath intensity profile of the typical urban channel is given by the following table:

Delay (μs)	Fractional power
0	0.189
0.2	0.379
0.5	0.239
1.6	0.095
2.5	0.061
5.0	0.037

1. Suppose that a signal of the 30 kHz bandwidth is transmitted over this channel. Is equalization required?
2. What about a 30 MHz signal?

Please argue your answer.

II. Consider transmission of the modulated signal $s(t)$ with the power 10 mW, carrier frequency f_c, and bandwidth (Bw) 40 MHz over a radio channel with the amplitude frequency response shown in Fig. 1 [$H(f) = H(-f)$]. The two-sided power spectral density of channel noise is $N_o/2 = 10^{-12}$ W/Hz.

1. Define the frequency response of the zero-forcing equalizer.
2. For the defined parameters, find the SNR at the equalizer output.

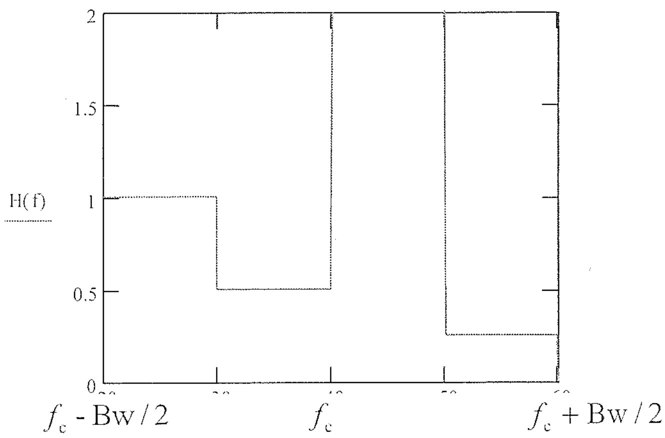

Fig.1

III. Consider a transmission of zero-mean symbols with the autocorrelation $\mathbf{R}_x$ over a MISO channel with L transmit antennas and channel gain vector $\mathbf{h} = [h_1, ..., h_L]^T$. The channel capacity for the fixed $\mathbf{h}$ is

$$C = \log_2\left(1 + \frac{\mathbf{h}^*\mathbf{R}_x\mathbf{h}}{N_0}\right)$$

where N_0 is the power spectral density of AWGN of the channel.

Consider a slow-fading channel where $\mathbf{h}$ is a multivariate complex-valued Gaussian variable with i.i.d. components. Consider 2 kinds of transmission: (a) the input symbols are correlated; (b) the input symbols are uncorrelated.

Given a total power constraint P compare for 2 above types of transmission:

1. The outage capacity.
2. The ergodic capacity

Hint: use the singular value decomposition of $\mathbf{R}_x$ taking into account that it is a symmetric matrix.

IV. Consider a communication system operating over a Nakagami-m SIMO channel with independent branches and employing maximal-ratio combining.

Give an explanation and prove (e.g. by applying the moment-generating function (MGF) – based approach) that increasing of the number of the branches improves the bit-error rate (BER) performance of the system.

Hint: An expression for the MGF of the power variable γ in Nakagami-m fading is:

$$M(s) = E\{e^{s\gamma}\} = \frac{1}{\left(1 - \frac{\bar{\gamma}}{m}s\right)^m}$$

where $\bar{\gamma}$ is the expectation of γ, and m is the Nakagami-m fading parameter.

V. Consider a MIMO system with the channel gain matrix $\mathbf{H} = \begin{bmatrix} 0.4 & 0.5 & 0.3 \\ 0.7 & 0.6 & 0.8 \end{bmatrix}$.

Consider the transmit precoding and receiver shaping.

1. Given a total power constraint P, find the maximal data rate that can be transmitted over the parallel SISO channels assuming M-QAM modulation on each channel with optimal power adaptation across the channels.

The target BER $P_b = 10^{-3}$. Assume that the constellation size is unrestricted , and $P_b \leq 0.2\exp[-1.5\gamma/(M-1)]$.