AALTO UNIVERSITY SCHOOL OF ELECTRICAL ENGINEERING
Exam January 12th, 2011
Department of Signal Processing and Acoustics
S-88.3105 Digital Signal Processing Systems (5 cr)

Write in each answer paper your name, department, student number, the course name and code, and the date. Number each paper you submit and denote the total no. of pages. 5 problems, 28 points total. Exam problems in English only. Please feel free to answer in Finnish or English. No additional material is allowed in the exam.

1. (4p) Explain *briefly* the following concepts:

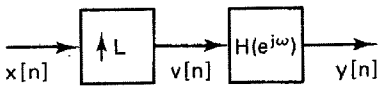
(a) Noble identities

(b) Cascaded integrator comb filter

(c) Commutator

(d) Tree-structured filter bank

2. (4p) The system below interpolates the sequence $x(n)$ by a factor L. Suppose that the linear filter has impulse response $h(n)$ such that $h(-n)$ and $h(n) = 0$ for $|n| > (RL - 1)$, where R and L are integers; i.e., the impulse response is symmetric and of length $(2RL - 1)$ samples.



a) In answering the following, do not be concerned about the causality of the system. It can be made causal by including some delay. Specifically, how much delay must be inserted to make the system causal?

b) What condition must $h(n)$ satisfy so that $y(n) = x(n/L)$ for $n = 0, \pm L, \pm 2L, \pm 3L, \cdots$?

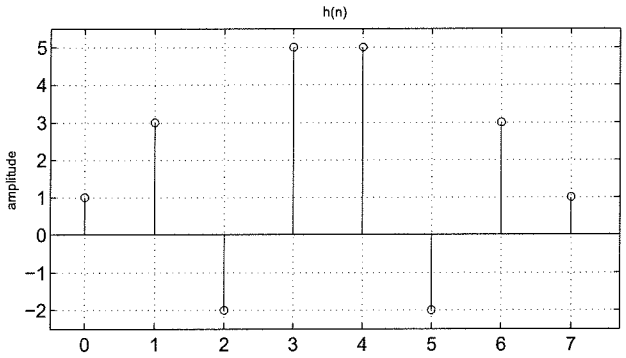
c) By exploiting the symmetry of the impulse response, show that each sample of $y(n)$ can be computed with no more than RL multiplications.

d) By taking advantage of the fact that multiplications by zero need not be done, show that only $2R$ multiplications per output sample are required.

3. (8p)

a) Explain the polyphase decomposition using mathematical formulas. Why may the polyphase decomposition be useful when designing a digital signal processing system?

b) Consider the impulse response of the filter $h(n)$ in the figure below:

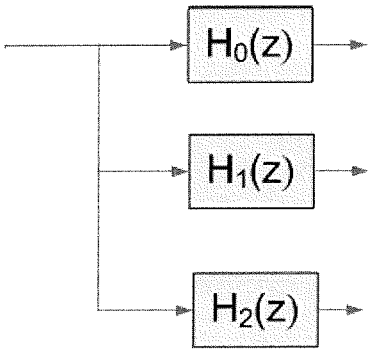


What can you say about amplitude and phase responses of $H(z)$?

c)
- Give the type I 2-branch polyphase decomposition of $H(z)$.
- Give the type II 3-branch polyphase decomposition of $H(z)$

Draw block diagrams for both realizations.

d) Let us consider the uniform analysis filter bank shown below



where $h_i(n) = h(n)e^{j2\pi ni/3}$. Determine the equivalent polyphase representation and draw its block diagram.

4. (8p) Show that in a perfect reconstruction linear-phase filter bank filters must be such that $H_0(z)H_1(-z)$ ($H_0(z)$ and $H_1(z)$ are the analysis filters) has an odd number of coefficients and that all but one of its odd powers of z must be zero.

5. (4p) The following equation holds for the scaling functions of wavelets.

$$\phi_{i,k}(t) = 2^{\frac{i}{2}}\phi(2^i t - k)$$

Draw the Haar scaling functions for $i, k = 0, 1, 2, 3$.