Vakioiden arvoja

Planckin vakio	$h = 6.6261 \cdot 10^{-34}$ Js
h/2π	$\hbar = 1.0546 \cdot 10^{-34}$ Js
valon nopeus tyhjiössä	$c = 2.9979 \cdot 10^{8}$ m/s
alkeisvaraus	$e = 1.6022 \cdot 10^{-19}$ C
tyhjiön permeabiliteetti	$\mu_0 = 4\pi \cdot 10^{-7}$ N/A^2
tyhjiön permittiivisyys	$\varepsilon_0 = 8.8542 \cdot 10^{-12}$ F/m
Boltzmannin vakio	$k = 1.3807 \cdot 10^{-23}$ J/K
elektronin lepomassa	$m_e = 9.1094 \cdot 10^{-31}$ kg
protonin lepomassa	$m_p = 1.6726 \cdot 10^{-27}$ kg
neutronin lepomassa	$m_n = 1.6749 \cdot 10^{-27}$ kg
Stefan-Boltzmannin vakio	$\sigma = 5.6705 \cdot 10^{-8}$ W/(m^{2}K^4)

Nabla sylinterikoordinaateissa

$$\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} + \frac{\hat{\varphi}}{r}\frac{\partial}{\partial \varphi} + \hat{k}\frac{\partial}{\partial z}$$

$$\vec{\nabla} \times \vec{A} = \left(\frac{1}{r}\frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z}\right)\hat{r} + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\hat{\varphi} + \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\varphi) - \frac{\partial A_r}{\partial \varphi}\right]\hat{k}$$

$$\vec{\nabla} \cdot \vec{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_r}{\partial z}$$

$$\nabla^2 \psi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \varphi^2} + \frac{\partial^2 \psi}{\partial z^2}$$

Nabla pallokoordinaateissa

$$\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} + \frac{\hat{\theta}}{r}\frac{\partial}{\partial \theta} + \frac{\hat{\varphi}}{r\sin\theta}\frac{\partial}{\partial \varphi}$$

$$\vec{\nabla} \times \vec{A} = \frac{1}{r\sin\theta}\left[\frac{\partial(\sin\theta A_\varphi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \varphi}\right]\hat{r} + \left[\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial \varphi} - \frac{1}{r}\frac{\partial (rA_\varphi)}{\partial r}\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial (rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta}\right]\hat{\varphi}$$

$$\vec{\nabla} \cdot \vec{A} = \frac{1}{r^2}\frac{\partial (r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial (\sin\theta A_\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial \varphi}$$

$$\nabla^2 \psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial \psi}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 \psi}{\partial \varphi^2}$$

$$= \frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial \psi}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 \psi}{\partial \varphi^2}$$

Integraaleja

$$\int \sin ax\, dx = -\frac{1}{a}\cos ax$$

$$\int \cos ax\, dx = \frac{1}{a}\sin ax$$

$$\int \sin^2 ax\, dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$$

$$\int \cos^2 ax\, dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

$$\int \sin^3 ax\, dx = -\frac{1}{a}\cos ax + \frac{1}{3a}\cos^3 ax$$

$$\int \cos^3 ax\, dx = \frac{1}{a}\sin ax - \frac{1}{3a}\sin^3 ax$$

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$$

$$\int_{-\infty}^{\infty} x^2 e^{-ax^2} dx = \frac{\sqrt{\pi}}{2a^{3/2}}$$

$$\int x \sin ax\, dx = \frac{1}{a^2}(\sin ax - ax\cos ax)$$

$$\int x \cos ax\, dx = \frac{1}{a^2}(\cos ax + ax\sin ax)$$

$$\int x^2 \sin ax\, dx = \frac{1}{a^3}(2\cos ax + 2ax\sin ax - a^2x^2\cos ax)$$

$$\int x^2 \cos ax\, dx = \frac{1}{a^3}(-2\sin ax + 2ax\cos ax + a^2x^2\sin ax)$$

$$\int x^n \sin ax\, dx = -\frac{1}{a}x^n\cos ax + \frac{n}{a}\int x^{n-1}\cos ax\, dx$$

$$\int x^n \cos ax\, dx = \frac{1}{a}x^n\sin ax - \frac{n}{a}\int x^{n-1}\sin ax\, dx$$

$$\int x \sin^2 ax\, dx = \frac{x^2}{4} - \frac{x\sin 2ax}{4a} - \frac{\cos 2ax}{8a^2}$$

$$\int x \cos^2 ax\, dx = \frac{x^2}{4} + \frac{x\sin 2ax}{4a} + \frac{\cos 2ax}{8a^2}$$

$$\int x^2 \sin^2(ax)dx = \frac{x^3}{6} - \frac{x\cos(2ax)}{4a^2} - \frac{(-1+2a^2x^2)\sin(2ax)}{8a^3}$$

Vektorilaskentaa

$$\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$$

$$(\vec{A} \times \vec{B}) \times \vec{C} = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{B} \cdot \vec{C})\vec{A}$$

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = (\vec{A} \times \vec{B}) \cdot \vec{C}$$

$$(\vec{A} \times \vec{B}) \cdot (\vec{C} \times \vec{D}) = \begin{vmatrix} \vec{A} \cdot \vec{C} & \vec{A} \cdot \vec{D} \\ \vec{B} \cdot \vec{C} & \vec{B} \cdot \vec{D} \end{vmatrix}$$

$$\vec{\nabla}(\psi\xi) = \xi\vec{\nabla}\psi + \psi\vec{\nabla}\xi$$

$$\vec{\nabla} \times (\psi\vec{A}) = \vec{\nabla}\psi \times \vec{A} + \psi\,\vec{\nabla} \times \vec{A}$$

$$\vec{\nabla} \cdot (\psi\vec{A}) = \vec{\nabla}\psi \cdot \vec{A} + \psi\vec{\nabla} \cdot \vec{A}$$

$$\vec{\nabla} \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\vec{\nabla} \times \vec{A}) - \vec{A} \cdot (\vec{\nabla} \times \vec{B})$$

$$\vec{\nabla}(\vec{A} \cdot \vec{B}) = (\vec{A} \cdot \vec{\nabla})\vec{B} + (\vec{B} \cdot \vec{\nabla})\vec{A} + \vec{A} \times (\vec{\nabla} \times \vec{B}) + \vec{B} \times (\vec{\nabla} \times \vec{A})$$

$$\vec{\nabla} \times (\vec{A} \times \vec{B}) = \vec{A}(\vec{\nabla} \cdot \vec{B}) - \vec{B}(\vec{\nabla} \cdot \vec{A}) + (\vec{B} \cdot \vec{\nabla})\vec{A} - (\vec{A} \cdot \vec{\nabla})\vec{B}$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2\vec{A}$$

Trigonometeriaa

1. $\sin^2\alpha + \cos^2\alpha = 1$ $\quad \tan\alpha = \frac{\sin\alpha}{\cos\alpha}$ $\quad \cot\alpha = \frac{\cos\alpha}{\sin\alpha} = \frac{1}{\tan\alpha}$

   $\frac{1}{\cos^2\alpha} = 1 + \tan^2\alpha$ $\quad \frac{1}{\sin^2\alpha} = 1 + \cot^2\alpha$

2. $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\quad \sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

   $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\quad \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

   $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$ $\quad \tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

   $\cot(\alpha+\beta) = \frac{\cot\alpha\cot\beta - 1}{\cot\alpha + \cot\beta}$ $\quad \cot(\alpha-\beta) = -\frac{\cot\alpha\cot\beta + 1}{\cot\alpha - \cot\beta}$

3. $\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$ $\quad \sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$

   $\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$ $\quad \cos\alpha - \cos\beta = -2\sin\frac{\alpha-\beta}{2}\sin\frac{\alpha+\beta}{2}$

4. $\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$

   $\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha-\beta) + \sin(\alpha+\beta)]$

   $\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$

5. $\sin 2\alpha = 2\sin\alpha\cos\alpha$ $\quad \sin^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2}$

   $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha$ $\quad \cos^2\frac{\alpha}{2} = \frac{1+\cos\alpha}{2}$

   $\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$ $\quad \tan\frac{\alpha}{2} = \frac{\sin\alpha}{1+\cos\alpha} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$

   $\cot 2\alpha = \frac{\cot^2\alpha - 1}{2\cot\alpha}$ $\quad \cot\frac{\alpha}{2} = \frac{\sin\alpha}{1-\cos\alpha} = \pm\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}$