# T-61.3040 Statistical Modeling of Signals: Exam 13.12.10

In the exam, you are allowed to have a calculator (non-programmable or memory emptied) and basic mathematical tables (no tables containing material directly associated with the course). For example, the book "BETA Mathematics Handbook for Science and Engineering" by Råde & Westergren contains material that is too directly associated with the course: therefore you are NOT allowed to have that book in the exam. The results of the exam will be announced eventually through the Noppa system. You can answer in English (preferred), Finnish or Swedish.

## 1 (max 10 points)

Explain the following topics *briefly but covering the most important properties*:

- LMS algorithm (2 points)
- ARCH model (3 points)
- Wiener filter (3 points)
- Wide-sense stationarity (WSS) (2 points)

## 2 (max 10 points)

You have observed the real-valued zero-mean WSS process $x(n)$ and you know the following values of the autocorrelation: $r_x(0) = 1$, $r_x(1) = \frac{2}{3}$, $r_x(2) = 0$, $r_x(3) = \frac{1}{3}$, $r_x(4) = 0$.

1. Form an optimal linear predictor, having two weights, where you predict the value $x(n)$ using observations: $x(n-1)$ and $x(n-2)$ . (4 points)
2. What is the mean-square prediction error of your predictor? How much smaller is the prediction error compared to the variance $\mathrm{Var}(x(n))$ of the process? (3 points)
3. Can you improve the prediction accuracy using a linear predictor with two weights, where you predict the value $x(n)$ using observations: $x(n-2)$ and $x(n-3)$ or observations $x(n-3)$ and $x(n-4)$ . (3 points)

# 3 (max 10 points)

Multiple choices questions. The following questions have different proposed answers. Only one of them is correct. You have to give your answer along with your confidence ("High" or "Low") for each answer. Grading for each of these multiple choices questions is then:

- +2 if answer is right and confidence is high
- +1 if answer is right and confidence is low
- 0 if answer is missing
- -1 if answer is wrong and confidence is low
- -2 if answer is wrong and confidence is high

Write on your answer sheet the correct answer (A, B, C, D,. . . ), along with the confidence you have (High, Low) for that question; e.g "A, Low" is a proper way of answering a question. Missing confidence for a question will be treated as "Low". Total score for this question is between 0 and 10 (total score of the exam is on 40). No need to justify your answers. The total of points is at least zero (you cannot get a negative score).

1. We use the covariance method to estimate the autocorrelation matrix. The estimate is (A) Unbiased and Toeplitz, (B) Biased and Toeplitz, (C) Unbiased and not Toeplitz, (D) Biased and not Toeplitz, (E) none of the previous answers is correct.

2. If $x(n) = d(n) + v(n)$, $x(n)$ is the measured signal, $v(n)$ is a white noise with unknown variance, $d(n)$ is the desired signal that we want to evaluate using a FIR-filter, $x(n)$ and the desired signal $d(n)$ are jointly WSS, $d(n)$ and the white noise $v(n)$ are uncorrelated then: (A) an optimal FIR-filter cannot be defined, (B) An optimal FIR-filter can be defined in theory but can never be solved, (C) An optimal FIR-filter can always be solved (D) An optimal FIR-filter can be solved if $r_d$ is known, (E) none of the previous answers is correct.

3. A periodogram and the Bartlett method are used to estimate the Power Spectrum of a WSS process $x(n)$, we compare the performances of the Bartlett method to the performances obtained with the periodogram as a baseline: (A) The variance is larger and the resolution is worse, (B) The variance is smaller and the resolution is better, (C) The variance is smaller and the resolution is worse, (D) The variance is larger and the resolution is better, (E) none of the previous answers is correct

4. The Pisarenko method is used to estimate the Power Spectrum of a WSS process $x(n)$. The pseudo-spectrum is calculated and (A) the pseudo-spectrum is a very good estimate of the Power Spectrum (B) The pseudo-spectrum is equal to zero for the main frequencies of the process $x(n)$,

(C) The pseudo-spectrum is infinite for the main frequencies of the process $x(n)$, (D) in the the pseudo-spectrum, some peaks appear for the main frequency of the process $x(n)$, (E) in the the pseudo-spectrum, some peaks appear for the main frequencies of the process $x(n)$, (F) none of the previous answers is correct.

5. We define a process $x(n) \sim \begin{cases} N(0,1) & \text{if} \quad x(n-1) > 0 \\ N(0,2) & \text{if} \quad x(n-1) \leq 0 \end{cases}$. The conditional variance $\text{var}\,(x(n) \mid x(n-1))$ is (A) always equal to the variance $\text{var}\,(x(n))$, (B) always smaller than the variance $\text{var}\,(x(n))$, (C) always larger than the variance $\text{var}\,(x(n))$, (D) always negative, (E) none of the previous answers is correct.

# 4 (max 10 points)

You have measured the following observations from the real-valued zero-mean WSS process $x(n)$: $x(0) = 1$, $x(1) = 0$, $x(2) = -\frac{1}{2}$, $x(3) = 0$, $x(4) = \frac{1}{4}$, $x(5) = 0$. You want to estimate a linear predictor with two weights, which predicts the value $x(n)$ based on the values $x(n-1)$ and $x(n-2)$. You want to use the LMS algorithm to estimate the predictor.

1. You want the LMS algorithm to converge in the mean. Choose a step size so that this happens; estimate the required information and make any necessary assumptions. A wise choice for the step size can maybe simplify the resolution of the next questions. (4 points)

2. Run the LMS algorithm for three update steps starting from n=2 and multipliers $\mathbf{w} = \begin{bmatrix} 1, & 1 \end{bmatrix}^T$. (4 points)

3. Assume that we know the mean-square prediction error (MSE) of an AR(2) model optimal for this process, the value of the MSE is $s_{min}$. Assume that we have run the LMS algorithm ; in run 1 we use a step size that is one thousandth of the step size you chose in part 1.; in run 2 we use one tenth thousands of the step size you chose in 1. Assume that in both runs the algorithm was run for a very long time, and the algorithm had converged in the mean. In both runs, after the convergence in the mean, the MSE of the LMS algorithm is still on average larger than $s_{min}$; explain why this is so. Denote the average MSE achieved in run 1 by $s_1$ and the average MSE achieved in run 2 by $s_2$; estimate (approximate) the ratio $(s_2 - s_{min}) \,/\, (s_1 - s_{min})$. (2 points)