

Mat-1.1020 Mathematics L2

Intermediate examination 1 21.02.2011

Please fill in clearly *on every sheet* the data on you and the examination. On *Examination code* mark course code, title and text mid-term or final examination. Study programmes are ARK, AUT, BIO, EST, ENE, GMA, INF, KEM, KJO, KTA, KON, MAK, MAR, PUU, RAK, TFY, TIK, TLT, TUO, YHD.

Calculators are not allowed. Examination time 3h.

1. Evaluate (exactly) the following integrals:

$$\text{a) } \int_0^{\pi/2} \cos^3 x \, dx \quad \text{b) } \int_0^1 \ln x \, dx \quad \text{c) } \int_0^{\infty} \frac{1}{e^x + 1} \, dx$$

2. Concerning the numbers

$$s_n = \sum_{k=1}^n \frac{1}{k^{5/4}}$$

we know that $s_n = 4.1951068 \dots$ when $n = 9999$. Based on this information evaluate the limit $s = \lim_n s_n$ with five significant digits. Justify the accuracy!

3. Concerning function $f(x)$ we know that $f(x)$ is continuously differentiable on the interval $[0, \infty)$, $f(0) = 1$, $f'(0) > 0$ and

$$1 + A(x) = \frac{1}{[f'(x)]^2}, \quad x > 0,$$

where $A(x)$ is the area between the curve $y = f(x)$ and the x -axis on the interval $[0, x]$. For which value of x is $f(x) = 2$?

4. Solve:

$$\text{a) } x^2 y' = x^2 + xy + y^2 \quad \text{b) } y'' + 3y' + 2y = x + e^{-x}$$