Aalto University
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EXAM
9.3.2011

S-72.3410 Coding Methods

1. (6p.) Consider the convolutional encoder with the transfer function matrix

$$\mathrm{G}(D) = [1 + D^2 \quad 1 + D + D^2].$$

Assume that this encoder is used over a memoryless binary symmetric channel. Find the maximum-likelihood codeword corresponding to the following received sequence:

$$\mathbf{r} = (10, 11, 11, 11, 01, 00).$$

2. (6p.) Construct systematic parity-check and generator matrices for a (13,10) 3-ary Hamming code (i.e. with code symbols taken from GF(3)).

3. (a) (2p.) Is it possible to construct a binary cyclic code with length 17 and dimension 13? What about a 4-ary cyclic code of length 17 and dimension 13? Justify your answers.

   (b) (2p.) Find the generator polynomial of a two-error-correcting binary cyclic code of length 255.

   (c) (2p.) Find the generator polynomial of a two-error-correcting 256-ary cyclic code of length 255. Try to express each coefficient of the polynomial as a power of a primitive element of GF(256).

4. (a) (3p.) A fountain code is used to transmit four packets (two bits each) of data, B_1, B_2, B_3, and B_4. Assume that the transmitted packets are B_1+B_2, B_2+B_3, $B_3 + B_4$, $B_1 + B_4$, $B_1 + B_3 + B_4$, and that the receiver gets the packets 00, 01, 01, 00, and 11 (in the same order). Determine the original packets B_i, $1 \leq i \leq 4$.

   (b) (3p.) Draw a Tanner graph corresponding to the parity-check matrix you obtained in Problem 2.