Aalto University
School of Science and Engineering
Department of Communications and Networking
S-38.1146 Introduction to Performance Analysis, Autumn 2010

Examination
27.10.2010
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Please answer to all five (5) questions

1. Assume that X is exponentially distributed with parameter λ. One of the claims below is the memoryless property of the exponential distribution and other claims are erroneous:

$$P\{X \geq i+j\} + P\{X \geq i\} = P\{X \geq j\}, \text{ for all } i,j \geq 0, \quad (a)$$
$$P\{X \geq i+j \,|\, X \geq i\} = P\{X \geq j\}, \text{ for all } i,j \geq 0, \quad (b)$$
$$P\{X \geq i+j \text{ and } X \geq i\} = P\{X \geq j\}, \text{ for all } i,j \geq 0. \quad (c)$$

   (a) Which of the claims, a), b), or c) is the memoryless property?
   (b) Derive the memoryless property for the exponential distribution, for which $P\{X \geq k\} = e^{-\lambda k}$.
   (c) Let us assume that X models the length (=holding time) of a telephone call, and $\lambda = 1/3$ (1/min). If the phone call has already lasted 2 minutes, what is the expectation of the remaining call holding time?

2. Consider a system with 6 parallel servers and 4 waiting places. The average service time is 3 minutes. Customers are served in their arrival order. Assume an excessive arrival stream, that is, every time a customer leaves the system, a new customer arrives immediately. Therefore the system is always full. What is the average time a customer spends in the system?

3. Consider elastic data traffic carried by a 10-Mbps link in a packet switched network. Use a pure sharing system model with a single server. New flows arrive according to a Poisson process at rate 9 flows per second, and the sizes of files to be transferred are independently and exponentially distributed with mean 1 Mbit. Let $X(t)$ denote the number of ongoing flows at time t.

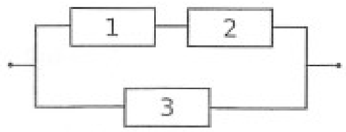
   (a) What is the traffic load?
   (b) Derive the equilibrium distribution of $X(t)$.
   (c) What is the throughput of a flow?

4. Consider the M/M/2/3 model with mean customer interarrival time of $1/\lambda$ time units and mean service time of $1/\mu$ time units. Let $X(t)$ denote the number of customers in the system at time t, which is a Markov process.

   (a) Draw the state transition diagram of $X(t)$.
   (b) Derive the equilibrium distribution of $X(t)$].
   (c) Assume that $\lambda = \mu$. What is the probability that an arriving customer is lost?

Last question on the other side of the paper

5. Consider the following system of independent components:



(a) What is the structure function $\phi(x)$ of the depicted system?

(b) If you further assume that the components are repairable, with independent failure and repair rates, with mean times given by the table below (mean time to failure and mean downtime) , what is the average availability of the system?

component	$MTTF_i$	MDT_i
1	4	1
2	2	2
3	2	1