Write in each answer paper your name, department, student number, the course name and code, and the date. Number each paper you submit and denote the total no. of pages. 5 problems, 30 points total. Papers in English only. The BETA mathematical tables can be utilized – you can borrow a copy from the exam supervisor if you do not have your own. A basic calculator can be used (no memory, no graphics).

The homework bonus will be valid for possible future exams too.

1. (1p each) Define and describe *briefly* (2..3 lines of text) the following concepts:

a) DFE
b) Viterbi equalizer
c) Echo cancellation
d) OFDM
e) Root-raised cosine filter
f) ZF equalizer

2. Adaptive equalizers (6p)

Let us consider a discrete-time model for a communication system in a linear channel (sampled at the symbol rate). The received signal samples $r(k)$ are filtered by an N-tap FIR filter (equalizer). The equalizer output $y(k)$ can be expressed as

$$y(k) = \mathbf{h}_{\mathrm{R}}^{\mathrm{T}} \mathbf{r}(k) \tag{1}$$

where $\mathbf{h}_{\mathrm{R}}$ and $\mathbf{r}$ are N-dimensional column vectors. Draw the receiver block diagram and derive the MSE gradient (MSEG) adaptive algorithm to update the equalizer coefficients. What is the optimal equalizer solution and in what conditions does the adaptive algorithm reach it? Discuss the convergence issues too.

3. Matched filters (6p)

a) (2p) Define the matched filter (MF) concept in an AWGN channel. Give the solution in both time-domain and frequency-domain forms.

b) (2p) Consider a discrete-time receive filter $h_{\mathrm{R}}(k)$ and its frequency response $H_{\mathrm{R}}(e^{j\omega k})$. Assume a simple discrete-time transmit filter:

$$h_{\mathrm{T}}(k) = 3\delta(k) - 2\delta(k+1) + \delta(k-2) \tag{2}$$

Find the matched-filter receive filter $h_{\mathrm{R}}(k)$ to $h_{\mathrm{T}}(k)$ in (2), and draw the impulse responses $h_{\mathrm{T}}(k)$ and $h_{\mathrm{R}}(k)$, both the ideal *noncausal* and *causal* versions.

c) (2p) Determine the pulse waveform $g(k)$ at the output of the receive filter either via convolution:

$$g(k) = h_{\mathrm{R}}(k) * h_{\mathrm{T}}(k) = \sum_{l=-\infty}^{\infty} h_{\mathrm{R}}(l) h_{\mathrm{T}}(k-l) \tag{3}$$

or in the frequency domain if you prefer. Plot $g(k)$.

4. Nyquist criterion (7p) [Bonus]:

The EDSL (=Extremely High Speed Digital Subscriber Line) company is facing a major crisis. The circuit design department has failed to produce a transmit filter that meets the Nyquist criterion. The CEO of the company ordered the chief designer (you) to dig up his SPIT1 course and solve the problem for next morning.

Following morning you suggest to use a transmit filter whose frequency response is constant over the frequency range $[-(1-\alpha)W_0, (1-\alpha)W_0]$ and goes linearly to zero in the range $[(1-\alpha)W_0, (1+\alpha)W_0]$ (and correspondingly at the negative frequencies).

a) (2p) Draw a sketch of the spectrum magnitude of the transmit filter. Also, convince the CEO that this transmit filter indeed meets the Nyquist criterion.

b) (3p) Solve for the corresponding signal waveform (time-domain impulse response).

c) (2p) What is the maximum α that can be used if the symbol rate $R = 1/T = 2400$ symbols/s and the total transmission bandwidth is 3000 Hz?

5. Channel capacity (8p) [Bonus]:

Consider the transmission of signal $x(t)$ over a linear channel with associated impulse response $c(t)$ and frequency response $C(f)$. The output waveform of the channel is then $r(t) = c(t)*x(t)$, where '*' denotes convolution. The output of the channel is thereafter corrupted by colored noise $n(t)$ with power spectral density (PSD) $S_n(f)$.

a) (4p) Solve for the optimum transmit power spectrum $S_{x,opt}(f)$ that maximizes the channel capacity C_{CH} when the total transmit power P_x is limited to 9 W. To help you solve the problem, the PSD of the noise $S_n(f)$ (in W/Hz) and the magnitude squared of the channel transfer function $|C(f)|^2$ are given by

$$S_n(f) = \begin{cases} 2.5, & |f| < 1\text{ Hz} \\ 1, & 1\text{ Hz} \le |f| < 6\text{ Hz} \\ \infty, & \text{otherwise} \end{cases} \qquad |C(f)|^2 = \begin{cases} 0.5, & |f| < 1\text{ Hz} \\ \frac{1}{|f|}, & |f| \ge 1\text{ Hz} \end{cases} \tag{4}$$

b) (2p) Determine the channel capacity resulting from the optimized transmit power spectrum in a).

c) (2p) Provide the minimum transmit power that justifies the transmission in the whole frequency band $|f| < 6$ Hz.

Hint: The optimal power spectrum is obtained with the water-pouring theorem as:

$$S_{x,opt}(f) = L - S_n(f)/|C(f)|^2$$

whenever resulting $S_{x,opt}(f)$ is positive (zero otherwise) and the water-filling L is determined so that the total transmit power is limited, i.e.,

$$P_x = \int_{-\infty}^{\infty} S_{x,opt}(f)df$$

The optimal capacity is then obtained by integration: $C_{CH} = \frac{1}{2}\int_{-\infty}^{\infty} \log_2\left(1 + \frac{S_{x,opt}(f)|C(f)|^2}{S_n(f)}\right) df$