Tfy-0.3223 Statistical Physics
Spring 2011

Midterm Exam 1 23.3.2011

1. Define or explain the following as accurately and concisely as you can:

   (a) order parameter
   (b) phase space
   (c) phase of matter
   (d) thermodynamic limit
   (e) Liouville equation
   (f) phase transition

2. Use the microcanonical formalism to calculate the entropy $S(E, V, N)$ of a classical ideal gas.
   Hint:
   $$V_N(R) = \frac{\pi^{N/2}}{(N/2)\Gamma(N/2)} R^N$$

3. Consider an ideal gas of massless photons, with spin $S = 1$ and energy $E_i = \hbar\omega_i = \hbar c k_i$. Photons have two polarized spin states, i.e. degeneracy $g = 2$.

   (a) The number of photons in a closed system is not conserved. Use the canonical ensemble to show that as a consequence, the chemical potential of the gas is zero.
   (b) Show by using the grand canonical ensemble that the average occupation number of photons at state ω is given by
   $$n(\omega) = \frac{2}{e^{\beta\hbar\omega} - 1}$$

4. Consider an interacting (monoatomic) gas, whose Hamiltonian is given by
   $$H = \sum_{i=1}^{N} \left[\frac{\vec{p}_i^2}{2m} + \sum_{j=1}^{i-1} V(r_{ij}) \right],$$
   where $r_{ij} = |\vec{r}_i - \vec{r}_j|$ is the distance between atoms i and j, and $V(r_{ij})$ is their pair interaction potential.

(a) Show that the canonical partition function can be written as

$$Z_N(T,V) = \frac{V^N}{N!\lambda_T^{3N}} Q_N(T,V),$$

where $\lambda_T = \frac{h}{\sqrt{2\pi m k_B T}}$ is the thermal de Broglie wavelength. What is $Q_N(T,V)$?

(b) Define the Meyer functions $f_{ij} = f(r_{ij}) = e^{-\beta V(r_{ij})} - 1$. Show that with these

$$Q_N(T,V) = \frac{1}{V^N} \int d\vec{r}_1 \cdots \int d\vec{r}_N \prod_{i<j} (1 + f_{ij}).$$

(c) In the limit where $f_{ij} \ll 1$, show that Q_N can be approximated by

$$Q_N(T,V) \approx 1 + \frac{N(N-1)}{2V^2} \int d\vec{r}_1 \int d\vec{r}_2 f(|\vec{r}_1 - \vec{r}_2|).$$

When is this a reasonable approximation? Hint: Consider the coefficient in front of the next order term.

Recall: $\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$