Tfy-0.3223 Statistical Physics
Spring 2011

Midterm Exam 2 20.5.2011

1. (a) Explain why there cannot be a Bose-Einstein condensation in a photon gas.
   (b) Explain why the Debye model of lattice vibrations gives a different low-T limit for the heat capacity of a set of quantum harmonic oscillators.
   (c) Explain the difference between tracer and collective diffusion.

2. Use the canonical formalism to derive the heat capacity of a set of N 3D quantum harmonic oscillators. Show that it reduces to the classical result $C_N = 3Nk_B$ in the limit $T \to \infty$.

3. Consider a gas of N spin-zero bosons in a d-dimensional container of volume V with a dispersion relation

$$\epsilon_p = \alpha |\vec{p}|^s.$$

Grand canonical partition function is

$$Z = \mathrm{Tr}\; e^{-\beta(\hat{H}-\mu\hat{N})} = \prod_p \frac{1}{1-e^{-\beta(\epsilon_p-\mu)}}.$$

The allowed wavevector modes are $\vec{k} = \frac{2\pi}{L}(n_x, n_y, n_z)$, where n_x, n_y and n_z are integers.

   (a) Find expressions for the mean number of particles per unit volume in the ground state and the mean total number of particles in the excited states, in terms of the temperature and the fugacity. (4 p.)
   (b) Find conditions for s and d for which Bose-Einstein condensation takes place. (2 p.)

   Remember,

$$V_d = \frac{2\pi^{d/2}}{\Gamma\left(\frac{d}{2}\right) d} R^d,$$

$$g_n(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^n} = \frac{1}{\Gamma(n)} \int_0^{\infty} \frac{z x^{n-1}}{e^x - z} dx, \text{ where } z = e^{\beta\mu}.$$

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4. Consider a 1D random walker that moves one step to the right with probability p_1, two steps to the right with probability p_2 and one step to the left with probability q on an infinite lattice with lattice spacing of L (see Fig. 1).

   (a) What is the condition that q, p_1 and p_2 must satisfy? (1 p.)

   (b) What is the condition that q, p_1 and p_2 must satisfy for there to be no particle drift (i.e. $\langle \Delta x \rangle$=0) after N steps ($N \to \infty$)? (2 p.)

   (c) Calculate the tracer diffusion coefficient when there is no drift. (3 p.)

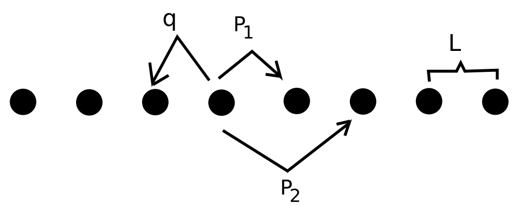

Figure 1: Random walk of question 4.