Tfy-0.3223 Statistical Physics
Spring 2011

Final Exam 26.5.2011

1. Define or explain the following as accurately and concisely as you can:
   (a) Phase space
   (b) Phase of matter
   (c) Bose-Einstein condensation
   (d) The difference between tracer and collective diffusion
   (e) Order parameter
   (f) Upper critical dimension

2. Consider a ring of 3 interacting spins in an external field. The Hamiltonian is given by

$$\mathcal{H}\left[\{S_i\}\right] = H\sum_{i=1}^{3} S_i - J\sum_{i=1}^{3} S_i S_{i+1},$$

where the spins can have values $S_i \in \{-1, 1\}$ for all $i = 1, 2, 3$, and the periodic boundary is given by $S_4 = S_1$. Calculate the partition function Z and the average magnetization M at temperature T. What is the relation between Z and M? The magnetization is defined by

$$M = \frac{1}{3}\sum_{i=1}^{3} S_i.$$

3. Consider the statistics of a non-interacting gas of particles on energy states E_ℓ. On any given state, there can be up to p particles, i.e. the allowed occupation numbers are $n_\ell = 0, 1, \ldots, p$. The total energy of the system is then $E = \sum_\ell n_\ell E_\ell$. (These hypothetical particles are called anyons.)
(a) Calculate the grand canonical partition function.
(b) Calculate the average occupation number $\langle n_\ell \rangle$.
(c) Consider the limits $p = 1$ and $p \to \infty$. What is $\langle n_\ell \rangle$ in these limits, and what are the corresponding statistics?

Hint: The finite geometric series is given by:

$$S_n = \sum_{k=0}^{n} r^k = \frac{1 - r^{n+1}}{1 - r}$$

TURN PAGE ⋙

4. Consider a surface of K adsorption sites in equilibrium with an ideal gas. Each of the K sites may be empty with energy $E_0 = 0$ or occupied by a gas particle with energy $E_1 < 0$. Calculate the fraction of occupied sites as a function of the chemical potential and temperature of the gas.

   Also, you may find the following mathematical identity useful:

$$\sum_{n=0}^{N} \binom{N}{n} x^n = (1+x)^N.$$