T-79.5104 Advanced Course in Computational Logic **Autumn term 2010/IN**
Exam May 26, 2011

1. (10p)

   a) Explain what is a classical tautology in modal logic. Is the formula $\Box\Box P \rightarrow ((\Box\Box P \rightarrow \neg\Box P) \rightarrow \Box\Box P)$ a classical tautology? Justify your claim.

   b) Define what it means that a frame logic **L** has the finite model property.

2. (10p) Determine using the tableau method whether the following claims hold. Give a counter-model based on the tableau when appropriate (P and Q are atomic propositions).

   a) $\{\} \models_{\mathbf{K4}} \{\neg\Diamond P\} \Longrightarrow \Box(\Box P \vee \neg\Diamond P)$, where **K4** is the class of transitive frames.

   b) There is a model based on a symmetric frame and a possible world in the model where all the formulas $\Diamond Q$ and $\Diamond(P \wedge \neg\Box Q)$ and $\Box\Diamond\Box\Diamond P$ are false.

3. (10p)

   a) Give a modal formula which is **D**-valid but not **K**-valid and give a model showing that this formula is not **K**-valid where **D** is the class of serial frames and **K** is the class of all frames.

   b) Consider a Hilbert-style proof system whose axioms are all classical tautologies and all formulas of the form $\Box(P \rightarrow Q) \rightarrow (\Box P \rightarrow \Box Q)$ and $\Box P \rightarrow \Box\neg\Box P$ and whose inference rules are the Modus Ponens and the necessitation rule.

      Define what it means that a Hilbert-style proof system is sound and complete for a given modal logic L and show that the proof system above is not sound for the modal logic **S4** where **S4** is the collection of reflexive and transitive frames.

4. (10p)

   a) Give the definitions of the following concepts in $\mathcal{ALC}$ in terms of the concept names `Bolt`, `Nut`, `Part` and role name `includes`:

      (i) A Crisp Part (a Part that is not a Bolt and not a Nut)

      (ii) A Complex Part (a Part that includes a Part).

   b) (i) Define what it means that a concept is subsumed by another with respect to a knowledge base in $\mathcal{ALC}$.

      (ii) Consider the knowledge base $(\mathcal{T}, \mathcal{A})$ where
      $\mathcal{T} = \{(B \sqcup \exists r.C) \sqsubseteq A\}$,
      $\mathcal{A} = \{a : (A \sqcup C)\}$,
      A, B, C are concept names, r is a role name, and a is an individual name.
      Study using the tableau algorithm for $\mathcal{ALC}$ whether $\exists r.A$ is subsumed by $\exists r.\neg C$ with respect to $(\mathcal{T}, \mathcal{A})$ and give a counter model when appropriate.

Properties of relation R:

Reflexive:	$\forall s(sRs)$
Symmetric:	$\forall s \forall t(sRt \rightarrow tRs)$
Serial:	$\forall s \exists t(sRt)$
Transitive:	$\forall s \forall t \forall u(sRt \wedge tRu \rightarrow sRu)$