Aalto University
School of Electrical Engineering
Department of Communications and Networking
S-38.1146 Introduction to Performance Analysis, Autumn 2011

Examination
2.9.2011
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Please answer to all five (5) questions

1. Assume that X is exponentially distributed with parameter λ. One of the claims below is the memoryless property of the exponential distribution and other claims are erroneous:

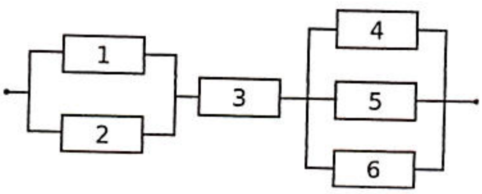
$$P\{X \geq i+j\} + P\{X \geq i\} = P\{X \geq j\}, \text{ for all } i, j \geq 0, \quad \text{(a)}$$
$$P\{X \geq i+j \mid X \geq i\} = P\{X \geq j\}, \text{ for all } i, j \geq 0, \quad \text{(b)}$$
$$P\{X \geq i+j \text{ and } X \geq i\} = P\{X \geq j\}, \text{ for all } i, j \geq 0. \quad \text{(c)}$$

   (a) Which of the claims, a), b), or c) is the memoryless property?
   (b) Derive the memoryless property for the exponential distribution, for which $P\{X \geq k\} = e^{-\lambda k}$.
   (c) Let us assume that X models the length (=holding time) of a telephone call, and $\lambda = 1/3$ (1/min). If the phone call has already lasted 2 minutes, what is the expectation of the remaining call holding time?

2. Consider a lossy queueing system with 2 parallel servers and 2 waiting places. The average interarrival time between two customers is 6 minutes, and the loss ratio is 10%. In addition, the average waiting time (before service) is 2 minute, and the average service time is 8 minutes.
   (a) What is the average number of waiting customers?
   (b) What is the average number of customers in service?

3. Consider the M/M/2 model with mean customer interarrival time of $1/\lambda$ time units and mean service time of $1/\mu$ time units. Let $X(t)$ denote the number of customers in the system at time t.
   (a) Draw the state transition diagram of Markov process $X(t)$.
   (b) Derive the equilibrium distribution of $X(t)$. Under which condition the system is stable?
   (c) Assumed that $\lambda = \mu$, what is the probability that the service of an arriving customer starts immediately upon the arrival (without any waiting)?
   (*Tip: You might need to use the formula for convergent geometric series in some form:* $\sum_{i=0}^{\infty} q^i = 1/(1-q), \quad |q| < 1$)

4. (a) Determine the structure function $\phi(\mathbf{x})$ of the structure of independent components in the reliability block diagram below.



   (b) If the components in above diagram are repairable, what is the availability of the above system? The availability of each of the components 1 and 2 is 2/3, the availability of component 3 is 1 and the availability of each of the components 4, 5 and 6 is 1/2?
   (c) Tell briefly in words what is **availability**? (That is, give some definiton of availability)

Last question on the other side of the paper

5. a) Assume the you can use a pseudo random number generator to easily generate samples of the random variable U that is uniformly distributed between $(0, 1)$, i.e., $U \sim \mathrm{U}(0, 1)$. Apply the inverse transform method to generate samples of the random variable X obeying $\mathrm{Exp}(\lambda)$ distribution (exponential distribution with mean $1/\lambda$).

   b) Again assume that you have a pseudo-random number generator to generate samples of $U \sim \mathrm{U}(0, 1)$. Give a pseudo code description for simulating a Poisson arrival process with intensity λ to count the number of arrivals between the time $[0, T]$.