Aalto University School of Electrical Engineering
Department of Communications and Networking
S-38.3143 Queueing Theory, Fall 2010

Examination
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Aalto

1. Consider a single-server queue. The system is empty at time 0. New customers arrive at times 1, 3, and 4. Their service times are 6, 3, and 2, respectively. For each of the three service disciplines given below, determine the departure times of all three customers:

   (a) FIFO, (b) LIFO-PR, (c) FB.

2. Consider a renewal sequence (T_n), where each T_n is uniformly distributed in the interval $(0, 1)$, $T_n \sim \mathrm{U}(0, 1)$. Let $T^s(t)$ denote the corresponding selected lifetime process. Utilizing the theory of regenerative processes, determine the mean $E[T^s]$ and the variance $D^2[T^s]$ of the steady-state selected lifetime distribution.

3. Consider an M/G/1 queue with two customer classes and deterministic service times for each class. New class-k customers arrive according to Poisson process with rate λ_k and their service times are equal to s_k. The service discipline is FIFO.

   (a) Are there any stability conditions? If so, specify them.

   (b) What is the mean steady-state waiting time when $\lambda_1 = \lambda_2 = 1/10$ (customers per time unit), $s_1 = 3$, and $s_2 = 4$ (time units)?

   (c) What is the mean steady-state queue length when $\lambda_1 = \lambda_2 = 1/10$ (customers per time unit), $s_1 = 3$, and $s_2 = 4$ (time units)?

4. Consider an M/G/1 queue with $\rho < 1$. The service discipline is the non-preemptive priority discipline with two customer classes. One of the customer classes has priority 1, and the other priority 2, where priority 1 refers to the higher priority. Prove that the steady-state mean waiting time E[W] is minimized when priority 1 is given to the user class with a smaller mean service time. You may utilize the fact that the steady-state mean waiting time for the two priority classes is given by

$$E[W_1] = \frac{E[R]}{1-\rho_1}, \qquad E[W_2] = \frac{E[R]}{(1-\rho_1)(1-\rho_1-\rho_2)}.$$

5. Consider a single-server queue that is operated using the Gittins index (GI) discipline. The Gittins index $G(a)$ is assumed to have the following properties:

   - $G(a) = 2 + a$ for $a \in [0, 2)$.
   - $G(a) = 8/a$ for $a \in [2, \infty)$.

   The system is empty at time 0. New customers arrive at times 1, 2, and 3. Their service times are 3, 6, and 5, respectively. Determine the departure times of all three customers.