## Examination

24.5.2011 Aalto

1. Consider a linear network with two links $j=1,2$. The long route $r=0$ uses both links, while the short routes $r=1,2$ use a single link ( $r$ ). Traffic consists of ordinary telephone calls that are IID and exponentially distributed with mean $1 / \mu$. New calls of class $r$ (using route $r$ ) arrive according to an independent Poisson process with intensity $\lambda_{r}$. Let $B_{r}$ denote the end-to-end blocking probability for class $r$. Assume now that $C_{1}=C_{2}=2$. By applying the Reduced Load Approximation method, give a system of equations from which an approximative value for $B_{0}$ can be solved (as a function of $a_{0}=\lambda_{0} / \mu, a_{1}=\lambda_{1} / \mu$, and $a_{2}=\lambda_{2} / \mu$ ).
2. Consider a closed queueing network with three customers and two nodes. The routing probabilities are $q_{11}=q_{12}=q_{21}=q_{22}=1 / 2$. The service times in both nodes are independent and exponentially distributed with mean 1 min .
(a) What is the mean number of customers in node 1 ?
(b) What is the mean sojourn time of a customer in node 1?
(c) What is the average customer flow through node 1 (in customers $/ \mathrm{min}$ )?
3. Consider a linear network with three links $j=1,2,3$. The long route $r=0$ uses all links, while the short routes $r=1,2,3$ use a single link $(r)$. Let $n_{r}>0$ denote the number of flows on route $r$. Assume that $n_{0}=2, n_{1}=1, n_{2}=2$, and $n_{3}=3$. All links have capacity $C=10$ Mbps , and all flows have the same access rate $s=10 \mathrm{Mbps}$. By applying the filling algorithm, determine the maxmin fair bandwidth shares $\left(x_{0}, x_{1}, x_{2}, x_{3}\right)$ for the flows on different routes.
4. Consider a base station in a cellular system utilizing channel-aware scheduling for the downlink transmissions consisting of elastic flows with mean size of $E[X]=100 \mathrm{Mbit}$. New flows arrive according to a Poisson process with mean interarrival time of $1 / \lambda=10 \mathrm{~s}$. Given that there are $n$ flows in the system at time $t$, the average rate at which the flows are served is assumed to be $c(n)=n r$, where $r=1 \mathrm{Mbit} / \mathrm{s}$. Use the PS queue with state-dependent service rate to model the system.
(a) What is the probability $P\{N=0\}$ that the system is empty?
(b) What is the mean number of flows $E[N]$ in the system?
(c) What is the mean delay $E[T]$ of a flow?
5. Consider a linear network with three links $j=1,2,3$. The long route $r=0$ uses all links (in the order $1 \rightarrow 2 \rightarrow 3$ ), while the short routes $r=1,2,3$ use a single link ( $r$ ). Let $n_{r}>0$ denote the number of flows on route $r$. Assume that $n_{0}=2, n_{1}=1, n_{2}=2$, and $n_{3}=3$. All links have capacity $C=10 \mathrm{Mbps}$, and all flows have the same access rate $a=10 \mathrm{Mbps}$. Determine the bandwidth shares ( $\varphi_{0}, \varphi_{1}, \varphi_{2}, \varphi_{3}$ ) for the flows on different routes "in the jungle" (without any end-to-end congestion control scheme) when the buffer management in the network nodes is tail dropping.
