Aalto University
School of Science and Technology
Department of Communications and Networking
S-38.1146 Introduction to Performance Analysis, Spring 2010

Examination
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Answers briefly:

1. (a) Lecture 2/23: $T \sim \text{Exp}(1/10)$. [3 points]
   (b) Lecture 2/24: $X \sim \text{Poisson}(1/2)$. [3 points]

2. (a) Birth-death process with state space $\{0, 1, 2, 3\}$ and state transition rates

$$q_{01} = q_{12} = q_{23} = \lambda, \quad q_{10} = \mu, \quad q_{21} = q_{32} = 2\mu. \quad \text{[2 points]}$$

   (b) We get the equilibrium distribution by applying the local balance equations together with the normalizing condition:

$$\pi_0 = \left(1 + \frac{\lambda}{\mu} + \frac{1}{2}\left(\frac{\lambda}{\mu}\right)^2 + \frac{1}{4}\left(\frac{\lambda}{\mu}\right)^3\right)^{-1},$$

$$\pi_1 = \pi_0 \frac{\lambda}{\mu},\ \pi_2 = \pi_0 \frac{1}{2}\left(\frac{\lambda}{\mu}\right)^2,\ \pi_3 = \pi_0 \frac{1}{4}\left(\frac{\lambda}{\mu}\right)^3. \quad \text{[3 points]}$$

   (c) Due to the PASTA property,

$$P\{\text{“loss”}\} = \pi_3 = 1/11 \approx 0.09 \quad \text{[1 point]}$$

3. Application of Little's formula/law/result (L5/67).
   (a) $E[X_\text{w}] = \lambda(1 - p_\text{loss})E[W] = \frac{1}{6} \cdot \frac{9}{10} \cdot 2 = 3/10 = 0.3$. [3 points]
   (b) $E[X_\text{s}] = \lambda(1 - p_\text{loss})E[S] = \frac{1}{6} \cdot \frac{9}{10} \cdot 8 = 6/5 = 1.2$. [3 points]

4. (a) Reliability block diagram is two blocks in series (L9/37). $\phi(\mathbf{x}) = x_1 x_2$ (L9/38). [2 points]
   (b) In total 4 states, probabilities can be calculated using 2 2-state models and using independence or directly from a 4-state model.

$$A = P\{\text{“both endpoints are up”}\} = \pi_3 = \frac{\mu_1}{\lambda_1 + \mu_1} \cdot \frac{\mu_2}{\lambda_2 + \mu_2} = \frac{\mu_1 \mu_2}{\lambda_1\lambda_2 + \lambda_1\mu_2 + \lambda_2\mu_1 + \mu_1\mu_2}$$

   [4 points]

5. (a) $X = \frac{1}{\lambda} \log U$. Some steps how to achieve this was required. [3 points]
   (b) Start at time 0, counter 0, use (a) to calculate time of next arrival, increment counter and time until time $> T$. [3 points]