

3. (6p) Lagrange polynomials are given in general form by

$$P_k(t) = \prod_{\substack{l=-N_1 \\ l \neq k}}^{N_2} \left(\frac{t - t_l}{t_k - t_l} \right)$$

- (a) Deduce Lagrange polynomials when $N_1 = N_2 = 1$.
 - (b) Give input-output relations in matrix form for an interpolator with a conversion factor of 3/2 using the polynomials above. Draw the block diagram as well.
 - (c) Describe the implementation of the 3/2 interpolator as a Farrow structure.
4. (6p) Draw a block diagram of the DFT polyphase analysis filter when the number of channels is M and the prototype filter is $H_0(z)$. Describe how the filter bank operates when the data rate of each output channel is $\frac{2}{M}$ of the rate of the input signal.
5. (8p) Given the lowpass analysis filter of a two-band FIR perfect reconstruction filter bank

$$H_0(z) = z^{-3} + az^{-2} + bz^{-1} + 2,$$

- (a) Determine the analysis and synthesis filters and discuss the class of filter banks to which they belong
- (b) Deduce the relation between a and b , assuming they are non-zero, in order to achieve perfect reconstruction