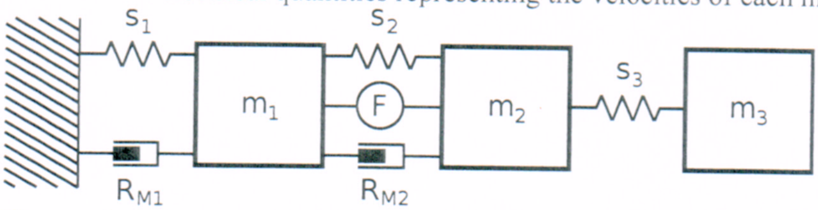
Important: If you have > 13/16 exercise points, mark clearly which problem you will replace with the exercise bonus.

Problem 1:
The equation below denotes the free vibration of a rectangular membrane, where z is displacement, x and y are the coordinates along the membrane, and L_x and L_y are the corresponding dimensions. Explain using your own words what is the physical interpretation (e.g. why is the term there, what concept does it represent) of
(a) n and m
(b) ω_{mn}
(c) the first two sine terms from the left
(d) the last term in parentheses (i.e. the term containing M and N)
(e) the double sum

$$z = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \sin\left(\frac{m\pi}{L_x}x\right) \sin\left(\frac{n\pi}{L_y}y\right) \left(M\sin(\omega_{mn}t) + N\cos(\omega_{mn}t)\right)$$

Problem 2:
Formulate the electrical equivalent circuit representation for the mechanical system illustrated below, using
(a) the impedance analogy
(b) the admittance analogy
What are the electrical quantities representing the velocities of each mass in (a) and (b)?

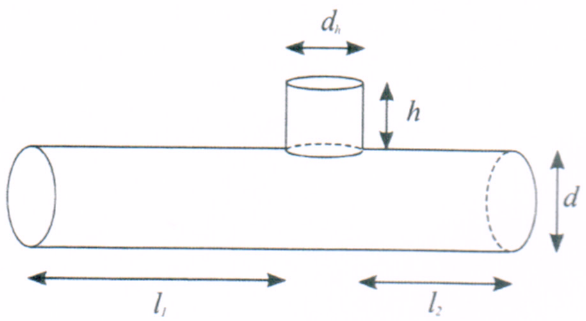


The symbol next to each mechanical element denotes the value of the associated parameter, e.g. the spring constant of the leftmost spring is s_1. Please remember to mark unambiguously the type of each electrical element and clearly denote their values.

Problem 3:
The natural frequency (eigenfrequency) of free undamped vibration of a system with one degree of freedom is 100 Hz. With an external force affecting, the maximum of the displacement of the damped system is obtained at the excitation frequency of 98 Hz. At which excitation frequency the maximum of the acceleration of the damped system will be obtained?

Problem 4:
Consider a small round electric motor which runs at 6000 rpm (rotations per minute) and has a physical volume of 4.2 liters. Due to a small imbalance in the rotor and a flexible mounting onto a chassis, the motor vibrates back and forth with an acceleration of 100 m/s^2. Evaluate the approximate acoustic power of the motor. You may consider the sound reflection effects negligible.

Problem 5:
Consider a cylindrical pipe of diameter d, as illustrated in the figure below. The pipe is open at both ends A and B, and it has a closed tube of diameter d_h as a side branch, which divides the pipe into segments of length l_1 and l_2. What is the value of the acoustic impedance seen from end A looking towards end B, when l_1=0.4m, l_2=0.1m, h=0.05m, d=0.04m, and d_h=0.01m, for a 200Hz signal?



Please fill in the electronic feedback form of the course, if you have not done it already. Thank you.

S-89.3310 Acoustics and Physics of Sound, equation sheet

Decibels:

$$L(\mathrm{dB\,re}\,P_0) = 10\log_{10}(P/P_0) \qquad \text{(Power quantities)}$$
$$L(\mathrm{dB\,re}\,A_0) = 20\log_{10}(A/A_0) \qquad \text{(Field quantities)}$$
$$P_0 = 1\mathrm{pW} \qquad \text{(Acoustic power)}$$
$$A_0 = 20\mu\mathrm{Pa} \qquad \text{(Sound pressure)}$$
$$v_0 = 1\mu\mathrm{m/s} \qquad \text{(Particle velocity)}$$

Constants:

$$c = 343\mathrm{m/s} \qquad \text{(Speed of sound, 20°, 1ATM)}$$
$$\rho = 1.2\mathrm{kg/m}^3 \qquad \text{(Density of air, 20°, 1ATM)}$$

Mechanical:

$$F_\mathrm{k} = -Kx \qquad \text{(Hooke)}$$
$$F_\mathrm{m} = m\ddot{x} \qquad \text{(Newton)}$$

Trigonometric:

$$e^{ix} = \cos x + i\sin x \qquad \text{(Euler)}$$
$$\sin^2 x + \cos^2 x = 1$$
$$\tan x = \frac{\sin x}{\cos x}$$
$$\cot x = \frac{1}{\tan x}$$

Harmonic oscillation:

$$x_\mathrm{RMS} = x_\mathrm{peak}/\sqrt{2} \qquad \text{(Peak values vs. RMS values)}$$
$$\tilde{x} = \tilde{A}e^{i\omega_0 t} \qquad \text{(Displacement)}$$
$$\tilde{v} = \dot{\tilde{x}} = i\omega_0\tilde{A}e^{i\omega_0 t} \qquad \text{(Velocity)}$$
$$\tilde{a} = \dot{\tilde{v}} = \ddot{\tilde{x}} = -\omega_0^2\tilde{A}e^{i\omega_0 t} \qquad \text{(Acceleration)}$$

1DOF oscillator:

$$\ddot{x} + 2\alpha\dot{x} + \omega_0^2 x = f \qquad \text{(Damped, forced)}$$
$$\alpha = R_\mathrm{m}/(2m)$$
$$\omega_0 = \sqrt{K/m}$$
$$\omega_\mathrm{d} = \sqrt{\omega_0^2 - \alpha^2}$$

Ideal string:

$$y(x,t) = \sum_{n=1}^{\infty}\sin\left(\frac{n\pi x}{L}\right)\left[A_n\sin\left(\frac{n\pi ct}{L}\right) + B_n\cos\left(\frac{n\pi ct}{L}\right)\right] \qquad \text{(Bernoulli)}$$

$$A_n = \frac{2}{n\pi c}\int_0^L \dot{y}(x,0)\sin\left(\frac{n\pi x}{L}\right)dx$$
$$B_n = \frac{2}{L}\int_0^L y(x,0)\sin\left(\frac{n\pi x}{L}\right)dx$$

Lossy, stiff, driven string:

$$\ddot{y} - c^2 y'' + 2R(f)\dot{y} + \frac{EA\kappa^2}{\mu}y'''' = f(x,t) \qquad \text{(Wave equation)}$$

Eigenfrequencies:

$$f_{mn} = \frac{1}{2}\sqrt{\frac{T}{\sigma}}\sqrt{\left(\frac{m}{L_x}\right)^2 + \left(\frac{n}{L_y}\right)^2} \qquad \text{(Rectangular membrane)}$$
$$f_{mn} = \frac{c}{2\pi R}\beta_{mn} \qquad \text{(Circular membrane)}$$
$$f_{lmn} = \frac{c_0}{2}\sqrt{\frac{l^2}{a^2} + \frac{m^2}{b^2} + \frac{n^2}{c^2}} \qquad \text{(Rectangular enclosure)}$$

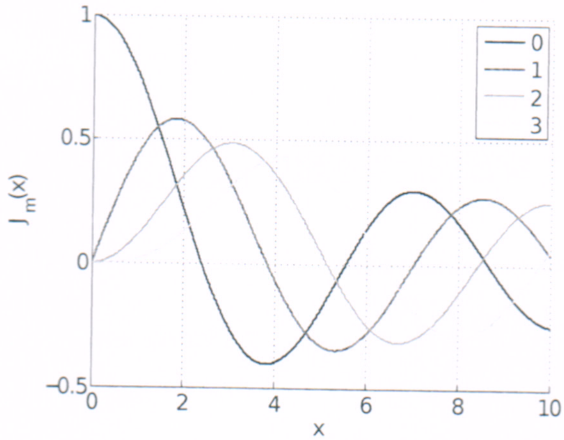


Figure 1: Some of the lowest-order Bessel functions

Modal densities:

$$n_\mathrm{a} = \frac{L}{2c_0} \qquad \text{(Axial modes)}$$
$$n_\mathrm{t} = \frac{\pi Af}{c_0^2} - \frac{L}{2c_0} \qquad \text{(Tangential modes)}$$
$$n_\mathrm{o} = \frac{4\pi f^2 V}{c_0^3} - \frac{\pi Af}{c_0^2} \qquad \text{(Oblique modes)}$$
$$n_\mathrm{tot} \approx \frac{4\pi f^2 V}{c_0^3} \qquad \text{(High-frequency approximation)}$$

Intensity:

$$\mathbf{I} = p\mathbf{u} \qquad \text{(p and $\mathbf{u}$ RMS values)}$$

Plane wave:

$$p = \tilde{A}e^{-ikx}e^{i\omega t} \qquad \text{(Sound pressure)}$$
$$k = 2\pi/\lambda = \omega/c \qquad \text{(Wave number)}$$
$$R = \frac{z_{c2} - z_{c1}}{z_{c2} + z_{c1}} \qquad \text{(Reflection coefficient)}$$
$$T = \frac{2z_{c2}}{z_{c2} + z_{c1}} \qquad \text{(Transmission coefficient)}$$
$$T = 1 + R$$
$$\mathbf{I}_\mathrm{t} = (1 - |R|^2)\mathbf{I}_\mathrm{i} \qquad \text{(Intensity transmission)}$$
$$\theta_\mathrm{i} = \theta_\mathrm{r} \qquad \text{(Snell)}$$

Characteristic impedance at distance d from the boundary between two fluids:

$$z_\mathrm{d} = \left(\frac{z_{c1}}{\cos\theta_\mathrm{i}}\right)\frac{\left(\frac{z_{c2}}{\cos\theta_\mathrm{t}}\right) + i\left(\frac{z_{c1}}{\cos\theta_\mathrm{i}}\right)\tan(k_1 d\cos\theta_\mathrm{i})}{\left(\frac{z_{c1}}{\cos\theta_\mathrm{i}}\right) + i\left(\frac{z_{c2}}{\cos\theta_\mathrm{t}}\right)\tan(k_1 d\cos\theta_\mathrm{i})}$$

el. impedance	el. admittance	mechanical	acoustical
voltage U	current I	force F	pressure p
current I	voltage U	velocity v	vol. velocity Q
impedance Z	admittance Y	mech. imped. Z_{M}	ac. imped. Z_{A}
resistance R	conductance G	mech. res. R_{M}	ac. res. R_{A}
inductance L	capacitance C	mass m	ac. ind. L_{A}
capacitance C	inductance L	compliance $\frac{1}{K}$	ac. cap. C_{A}
series conn.	parallel conn.	common velocity	common vol. vel.
parallel conn.	series conn.	common force	common pressure

Table 1: Table of analogies

Derivation rules:

$$D\frac{f(x)}{g(x)} = \frac{g(x)Df(x) - f(x)Dg(x)}{g(x)^2} \quad \text{(Derivation of a fraction)}$$

Spherical sources and -waves:

$$A = 4\pi R^2 \quad \text{(Surface area of sphere)}$$

$$V = \frac{4\pi R^3}{3} \quad \text{(Sphere volume)}$$

$$q_0 = \oint_A v_0 dA = 4\pi R^2 v_0 \quad \text{(Source strength)}$$

$$\tilde{p} = \frac{i\omega\rho q_0}{4\pi r}\frac{1}{1+ikR}e^{-ik(r-R)} \quad \text{(Pressure at distance } r\text{)}$$

$$z_{\mathrm{s}} = \frac{p}{u} = \rho c\left(\frac{ikr}{1+ikr}\right) \quad \text{(Characteristic impedance)}$$

$$P = \frac{|q_0|^2\rho c k^2}{8\pi}\left(\frac{1}{1+kR^2}\right) \quad \text{(Acoustic power)}$$

$$z_{\mathrm{mrad}} = 4\pi R^2\rho c\left(\frac{(kR)^2}{1+(kR)^2} + i\frac{kR}{1+(kR)^2}\right) \quad \text{(Radiation impedance)}$$

$$\tilde{p}(r,\theta) = \frac{\omega^2\rho}{4\pi c r}\left(1+\frac{1}{ikr}\right)e^{-ikr}\mu\cos\theta \quad \text{(Sound pressure at } r \text{ (dipole))}$$

$$\mu = q_0 d \quad \text{(Dipole moment)}$$

$$P = \frac{\omega^4\rho\mu^2}{24\pi c^3} \quad \text{(Acoustic power (dipole))}$$

Piston source:

$$\tilde{p}(r,\theta) = \frac{i\rho\omega R^2\tilde{u}_{\mathrm{n}}}{2r}\left[\frac{2J_1(kR\sin\theta)}{kR\sin\theta}\right]e^{-ikr} \quad \text{(Sound pressure)}$$

$$z_{\mathrm{mrad}} \approx \rho c\pi R^2\left[\frac{(kR)^2}{2} + i\frac{8kR}{3\pi}\right] \quad \text{(Radiation impedance } (kR \ll 1)\text{)}$$

Radiator groups:

$$\tilde{p}(\theta,r) \approx \left(\frac{i\omega\rho q_0}{4\pi r}\right)e^{-ikr}\left[\frac{\sin\left(\frac{N\pi d}{\lambda}\cos\theta\right)}{\sin\left(\frac{\pi d}{\lambda}\cos\theta\right)}\right] \quad \text{(Sound pressure (equal-phase sources)))}$$

Pipes:

$$z_{\mathrm{a}} = \frac{p}{q} = \frac{p}{uA} = \frac{z_{\mathrm{c}}}{A} \quad \text{(Acoustic impedance)}$$

$$z_{\mathrm{a1}} = \frac{\rho c}{A}\frac{z_{\mathrm{a2}}A\cos(kl) + i\rho c\sin(kl)}{iz_{\mathrm{a2}}A\sin(kl) + \rho c\cos(kl)} \quad \text{(} z_{\mathrm{a}} \text{ of a pipe, from end 1} \to \text{end 2)}$$

$$R = \frac{z_{\mathrm{a2}} - z_{\mathrm{a1}}}{z_{\mathrm{a2}} + z_{\mathrm{a1}}} \quad \text{(Reflection)}$$

$$T = \frac{2z_{\mathrm{a2}}}{z_{\mathrm{a2}} + z_{\mathrm{a1}}} \quad \text{(Transmission)}$$

$$T = R + 1$$

$$\frac{I_{\mathrm{t}}}{I_{\mathrm{i}}} = \frac{4}{4\cos^2(kl) + (\frac{A_1}{A_2} + \frac{A_2}{A_1})^2\sin^2(kl)} \quad \text{(Expansion chamber)}$$

$$\triangle = \begin{cases} 0.61R & \text{when } l > 0 \\ 0.85R & \text{when } l = 0 \end{cases} \quad \text{(End correction)}$$

Mechanical → electric system, impedance analogy:

1. make a circuit loop for each mass
2. make a circuit loop for each generator not connected to a mass
3. into each loop: add the electrical version of the directly involved components in series
4. connect different loops by combining their shared elements

Mechanical → electric system, admittance analogy:

1. for each mass, make circuit node with a grounded capacitor
2. make a grounded node for each generator not connected to a mass, and a grounded node for a rigid body
3. into each node: add the electrical version of the directly involved components in parallel
4. connect different node branches by combining their shared elements

Star-to-triangle transform:

1. inside each loop, insert a reference point
2. insert also a reference point outside the circuit
3. connect the points with lines
4. re-draw the connected pattern next to the circuit
5. for each line, change the intersecting series connection to parallel and switch the components into their dual versions