Aalto University School of Science, Department of Information and Computer Science TJ

T-79.3001 Logic in Computer Science: Foundations

Examination, October 26, 2011

Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 (10p)

(a) Define the following concepts: *model*, *complete proof system*, and *decidability*. (3 × 2p)

(b) What is meant by notation $\Sigma \models \phi$? Prove in detail that if $\Sigma \models \phi$ and $\Sigma \models \psi$, then $\Sigma \models \phi \wedge \psi$. (4$p$)

Assignment 2 (10p) Prove the following claims using semantic tableaux:

(a) $\models (A \to D) \vee (B \to D) \vee (C \to D) \to (A \wedge B \wedge C \to D)$.

(b) $\models \forall x \exists y (P(x) \wedge Q(y)) \to \exists y \forall x (P(x) \wedge Q(y))$.

Tableau proofs must contain all intermediary steps !!!

Assignment 3 (10p) Derive a Prenex normal form and a clausal form (i.e., a set of clauses S) for the sentence $\neg(\exists x(A(x) \vee B(x) \vee C(x)) \to \exists x A(x) \vee \exists y B(y) \vee \exists z C(z))$.

Make S as simple as possible. Prove that S is unsatisfiable using resolution.

Assignment 4 (10p) Let us represent natural numbers $0, 1, 2, \ldots$ using ground terms 0, $s(0)$, $s(s(0))$,... built of a constant symbol 0 and a function symbol s which is interpreted as the function $s(x) = x + 1$ for natural numbers x.

(a) Define a predicate $M(x, y, z)$ = "number y belongs to the interval $x, \ldots, z$ including end points" using sentences of predicate logic so that your definition covers all natural numbers (represented in the way explained above).

(b) Provide a counter model, on the basis of which your definition does not entail

$$\forall x \forall y \forall z (M(x, y, z) \to M(z, y, x)).$$

Assignment 5 (10p)

(a) Derive for the program `if(x<y) then {z=y-x} else {z=x-y}` the *weakest precondition* strarting from the *postcondition*

`(z>0)`. (4p)

(b) Consider the following program `Middle`:

```
z=x ; v=y ; while(!(z==v)) {z=z+1 ; v=v-1}.
```

Use weakest preconditions and a suitable invariant (6p) to establish

$\models_p$ `[true] Middle [2*v==x+y]`.

The name of the course, the course code, the date, your name, your student number, and your signature must appear on every sheet of your answers.