Aalto University School of Science, Department of Information and Computer Science TJ

T-79.5104 Advanced Course in Computational Logic

Examination, December 16, 2011

Assignment 1 (Max. 10p)

(a) Define formally *generated submodels*. What is the fundamental property expected from them?

(b) Given a set of frames **L**, define the *logical consequence* relation $\Sigma \models_{\mathbf{L}} \Upsilon \Longrightarrow P$. Describe in which sense this relation is monotonic.

Assignment 2 (Max. 10p) Use the *tableau method* to determine whether the following claims hold. Give a counter-model based on the tableau if appropriate. (Symbols P and Q denote atomic propositions below.)

(a) The formula $\neg((\Box P \wedge \Box\neg P) \vee (\Diamond P \wedge \Diamond\Box\neg P))$ is **KT4B**-valid where **KT4B** is the set of reflexive, transitive, and symmetric frames.

(b) There is a model based on a transitive and serial frame and a possible world in the model where the formula $\Diamond(P \vee \Diamond\Box\Box P)$ is true but the formula $\Diamond P$ is not.

Assignment 3 (Max. 10p)

(a) Show that the following two modal logics coincide: **KB4** based on symmetric and transitive frames and **KB5** based on symmetric and Euclidean frames.

(b) Suppose that you have devised a sound and complete proof method for the frame logic **S5** (reflexive, symmetric and transitive frames). What does this mean? What can be stated about the soundness and completeness of the method with respect to **KD45** (based on serial, transitive, and Euclidean frames)?

Assignment 4 (Max. 10p)

(a) Define the following concepts in $\mathcal{ALC}$ extended by *inverse roles* using the concept name `Worker` and the role name `supervises`:

1. A manager (a worker who supervises at least one worker)
2. A director general (a manager who supervises only mangers and is not supervised by any worker)

(b) Consider a knowledge base $(\mathcal{T},\mathcal{A})$ having TBox $\mathcal{T} = \{A \sqsubseteq C, B \sqsubseteq C\}$ and ABox $\mathcal{A} = \{a : (\exists r.A \sqcup \exists r.B)\}$ where A, B, and C are concept names, r is a role name, and a is an individual name.

Use the tableau algorithm for $\mathcal{ALC}$ to study whether the KB $(\mathcal{T},\mathcal{A})$ entails that the individual a is an instance of the concept $(\exists r.C)$ and give a counter model if appropriate.

Properties of a relation R:				
	Reflexive:	$\forall s(sRs)$	Serial:	$\forall s\exists t(sRt)$
	Symmetric:	$\forall s\forall t(sRt \rightarrow tRs)$	Euclidean:	$\forall s\forall t\forall u(sRt \wedge sRu \rightarrow tRu)$
	Transitive:	$\forall s\forall t\forall u(sRt \wedge tRu \rightarrow sRu)$		

The name of the course, the course code, the date, your name, your student identifier, and your signature must appear on every sheet of your answers.