## AS-74. 3125  Optimal, Adaptive and Robust Control
## Exam 4. 1. 2012

The questions are available only in English. You can answer in Finnish, Swedish or English. The final grade is given when both the examination and the homework problems have been evaluated.

5 problems.

1.  A process has the transfer function

$$G(s) = \frac{b}{s(s+1)}$$

where $b$ is a time-varying parameter. The system is controlled by a proportional controller

$$u(t) = k\left[u_c(t) - y(t)\right]$$

where $u$, $u_c$, and $y$ are controller output, reference and process output signals. It is desirable to choose the feedback gain so that the closed-loop system has the transfer function

$$G_{cl}(s) = \frac{1}{s^2 + s + 1}$$

Construct a continuous-time indirect self-tuning algorithm for the system.

2.  The system

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -x_1(t) + \left[1 - x_1^2(t)\right]x_2(t) + u(t) \end{cases}$$

is to be controlled to minimize the performance measure

$$J = \int_0^1 \frac{1}{2}\left[2x_1^2(t) + x_2^2(t) + u^2(t)\right]dt$$

The initial and final states are given (fixed).

a.  Determine the co-state equations for the system.
b.  Determine the control that minimizes the Hamiltonian for:
   (i)     $u(t)$ not bounded

(ii) $\quad |u(t)| \le 0$

Note: in part **b** the solution can contain co-state variables i.e. you do not have to solve the co-state equations.

3. Consider the same process as in Problem 2

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -x_1(t) + \left[1 - x_1^2(t)\right] x_2(t) + u(t) \end{cases}$$

Use a suitable Lyapunov function and state conditions for asymptotic stability of the system. Then choose the possibly nonlinear control law for $u(t)$ to fulfill the equations. (In short you use Lyapuvov analysis to design stabilizing controller for the system.)

4. Explain what is meant by the *minimum time (optimal) problem*, formulate it and discuss its solution methods and solutions in general. Use formulas and figures where appropriate.

5. Explain shortly the following concepts (and their meaning in control)

   a. Calculus of Variations
   b. Direct and indirect adaptive algorithms
   c. Model reference adaptive control (draw a schematic picture)
   d. Kalman-Yakubovich lemma
   e. M$\Delta$ structure in robust control
   f. Hamilton-Jacobi-Bellman equation

**Hints:**

$$\begin{aligned} p^n y_f(t) &= p^n H_f(p) y(t) \\ &= \varphi^T(t)\theta \end{aligned}$$

$$\begin{aligned} \theta &= \left[a_1 \cdots a_n \ b_1 \cdots b_m\right]^T \\ \varphi^T(t) &= \left[-p^{n-1} y_f \cdots - y_f \ p^{m-1} u_f \cdots u_f\right] \\ &= \left[-p^{n-1} H_f(p) y \cdots - H_f(p) y \ p^{m-1} H_f(p) u \cdots H_f(p) u\right] \end{aligned}$$

$$\begin{aligned} \frac{d\hat{\theta}(t)}{dt} &= P(t)\varphi(t)e(t) \\ e(t) &= y(t) - \varphi^T(t)\hat{\theta}(t) \\ \frac{dP(t)}{dt} &= \alpha P(t) - P(t)\varphi(t)\varphi^T(t)P(t) \end{aligned}$$