

# S-72.2505 Digital Transmission Methods

## Exam 14. 12. 2011

All five tasks are evaluated and taken into account in the grading. The exam can be written in Finnish, Swedish or English.

This is a closed book exam

1. A number of transmitters use CDMA. The baseband signal bandwidth is  $B = 5\text{MHz}$  and the length of the spreading code is  $T = 1\text{ms}$ . Perfect sinc pulse shaping is assumed.

- What is the chip duration?
- What is the spreading factor?
- How many independent users can communicate by transmitting at the same time if they use different orthogonal spreading codes and each user utilizes a complex baseband signal?
- What would the data rate of one user be if QPSK signalling is used?

2. Calculate the delay spread, the rms delay spread and mean delay spread of the channel described in table below. What is the channel coherence bandwidth? Is the channel frequency selective or frequency flat for WCDMA, with a signal bandwidth of 3.84 MHz?

i	1	2	3	4
$\tau_i$ [ $\mu\text{s}$ ]	0	0.1	1	2
$P_i$ [dB]	0	0	-3	-6

3. Consider a transmission using two signals in the interval  $[0, T]$ . One transmission has value 1 between 0 and  $2T/3$  and is zero elsewhere and the other has value 1 between  $T/3$  and  $T$  and is zero otherwise.

$$s_1(t) = \begin{cases} 1 & t \in [0, \frac{2T}{3}] \\ 0 & t \in (\frac{2T}{3}, T] \end{cases} \quad s_2(t) = \begin{cases} 0 & t \in [0, \frac{T}{3}] \\ 1 & t \in [\frac{T}{3}, T] \end{cases}$$

- Use the Gram-Schmidt algorithm to derive a set of orthonormal basis functions for this signal set.
  - Give the vectors of 2D coordinates representing these signals given the computed orthonormal basis functions.
  - Show that the basis functions are orthogonal.
  - Draw the constellation diagram of the signal set if  $T = 1$ .
4. Consider a frequency flat channel with Gaussian noise. The average signal-to-noise ratio (SNR) is 12 dB.

- For sufficient performance an instantaneous SNR 7 dB is required. Determine the time fraction when the SNR is below this value in a Rayleigh fading channel.



- b) Determine the time fraction when the SNR is below 7 dB when two-branch diversity with selection combining is used. The diversity branches have the same average strength and their fading is independent.  
Hint: The distribution of the instantaneous signal-to-noise ratio for two-branch selection combining is

$$p(\gamma) = \frac{2}{\bar{\gamma}} e^{-\gamma/\bar{\gamma}} (1 - e^{-\gamma/\bar{\gamma}})$$

Here,  $\bar{\gamma}$  is the average SNR, and  $\gamma$  is the instantaneous SNR, both in the linear domain.

5. We have a system where packet data is transmitted using uncoded BPSK in blocks of length  $n = 100$ . The probability that a block is received erroneously (not all bits are correct) is

$$P_{\text{Block}}(\gamma, n) = 1 - [1 - P_{\text{Bit}}(\gamma)]^n$$

where  $\gamma$  is the Signal-to-Noise Ratio (SNR), and the bit error probability may be approximated by  $P_{\text{Bit}}(\gamma) = \frac{e^{-\gamma}}{2}$ . Throughput is the data rate of correctly received blocks,  $T = \log_2(M) \times (1 - P_{\text{Block}}(\gamma))$ , where  $M$  is the constellation size. We are interested in a target throughput  $T_{\text{target}} = 0.8$  [bps/Hz]

- What SNR is required to achieve the target throughput?
- What is the approximate bit-error probability at this SNR?
- If we would use a transmission method achieving Shannon's capacity, which SNR should we use to achieve this throughput?
- What is the loss in SNR [dB] from using uncoded BPSK to realize this transmission?