S-72.2505 Digital Transmission Methods

Exam 12.12. 2012

All five tasks are evaluated and taken into account in the grading. The exam can be written in Finnish, Swedish or English.

This is a closed book exam

1. The mobile radio channel can be roughly characterized by two numbers, the coherence time, and the coherence bandwidth.

   a) What is the relation of the coherence bandwidth to the delay spread of the channel?

   b) Using the coherence bandwidth and the system bandwidth, how do you define a frequency flat vs. frequency selective fading channel?

   c) What is the essential difference when communicating over a frequency selective as opposed to a frequency flat channel?

   d) What is the relation of the coherence time to the Doppler spread of the channel, and what is the relation of Doppler spread to speed?

   e) Using the coherence time and the symbol period, how do you define a slow vs. rapidly fading channel?

   f) What is the essential difference when communicating over rapid fading as opposed to slow fading?

   (Short answers to the six questions above are expected, at most a couple of sentences.)

2. The Fourier transform of the unit step function at time t', $U(t-t') = \begin{cases} 1 & \text{if } t \geq t' \\ 0 & \text{if } t \leq t' \end{cases}$ is

$$U(f) = \mathrm{e}^{2\pi \mathrm{j} f t'} \left(\frac{1}{2\pi \mathrm{j} f} + \pi \delta(f) \right) .$$

   Calculate the frequency response, i.e. the Fourier transform of the rectangular pulse $\mathrm{rect}(t) = U(t+\frac{T}{2}) - U(t-\frac{T}{2})$, and the power spectrum density. What can we conclude from this on the usability of the rectangular pulse for transmission in a bandlimited system?

3. The receiver receives a signal over two independent channels in signal space. The amplitudes of the wanted parts of the channels are $\mathbf{a} = [6\ 2]$, and the amplitude of the transmitted symbols are 1. In both channels the noise variance is the same, $\sigma_N^2 = 4$. We assume a real channel with real signals and real noise.

   a) Calculate the received SNR if the receiver uses only the first channel.

   b) Calculate the received SNR for maximum ratio combining (MRC).

   c) Calculate the received SNR for equal gain combining (EGC).

   d) Express the gain achieved while using MRC and EGC compared to using only the first channel as in a).

4. Consider two signal waveforms in the time interval interval $\mathcal{D} = [-1, 1]$. One of them should be the triangular function, the other, linearly independent from the first one, could be the constant signal:

$$\begin{aligned} \phi_1(t) &= \text{tri}(t) = 1 - |t| \\ \phi_2(t) &= \text{rect}(t/2) = 1_{[-1,1]} \end{aligned}$$

Use Gram-Schmidt orthonormalization to find two basis functions, where the first element of the basis is a triangular function.

5. We have a channel with two multipath components. The inter-arrival time of the paths is very small compared to the symbol length. The channel can be modelled as

$$h = a_1 e^{j\phi_1} + a_2 e^{j\phi_2} ,$$

where $a_1 = 1$ and $a_2 = 0.5$. The noise power spectral density is $N_0 = 0.1$.

a) Compute the signal-to-noise ratio (SNR) if the phases happen to be such that the components are summed together coherently in phase (, i.e. $\phi_1 = \phi_2$).

b) Compute the SNR if the phases happen to be such that the components are combined destructively (the phase difference between them is π).

c) Assume that half of the time the receiver sees the constructive and half of the time the destructive combination of the components. BPSK signaling is used. What is the average bit error ratio?

Hint: In AWGN with SNR γ, the error probability of BPSK is $Q(\sqrt{2\gamma})$. In your computation you can approximate the Q-function as $Q(x) \sim e^{-x^2}$.

$e_2 = \frac{u_2}{\|u_2\|}$

$e_2 = \phi_2(t) - \frac{\langle \phi_1, \phi_2 \rangle}{\|\phi_1\|^2} \phi_1(t)$

$\frac{3}{2} \div \sqrt{0.5}$

$= \frac{3}{2} \times \frac{1}{\sqrt{0.5}}$

$= \frac{3}{2\sqrt{0.5}}$