Aalto University School of Science, Department of Information and Computer Science TJ

T-79.3001 Logic in Computer Science: Foundations

Examination, January 9, 2012

Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 (10p)

(a) Define the following concepts: *proof*, *ground term*, and *modus ponens*. ($3 \times 2p$)

(b) What is meant by the notation $\phi \equiv \psi$?

Prove in detail that if $\phi \equiv \chi$ and $\psi \equiv \chi$, then $\phi \wedge \psi \equiv \chi$. (4$p$)

Assignment 2 (10p) Prove the following claims using semantic tableaux:

(a) $\models A \vee B \rightarrow ((A \rightarrow C) \rightarrow ((B \rightarrow D) \rightarrow C \vee D))$.

(b) $\{\forall x(P(x) \leftrightarrow \neg Q(x)), \forall y(Q(y) \leftrightarrow \neg R(y)), \forall z(R(z) \leftrightarrow \neg P(z))\} \models \forall x P(x)$.

Tableau proofs must contain all intermediary steps !!!

Assignment 3 (10p) Derive a Prenex normal form and a clausal form (i.e., a set of clauses S) for the sentence $\neg(\exists x(P(x) \vee \forall y Q(x,y)) \rightarrow \exists y(P(y) \vee Q(y,y)))$.

Make S as simple as possible. Prove that S is unsatisfiable using resolution.

Assignment 4 (10p) Let us represent natural numbers $0, 1, 2, \ldots$ using ground terms 0, $s(0)$, $s(s(0))$,... built of a constant symbol 0 and a function symbol s which is interpreted as the function $s(x) = x + 1$ for natural numbers x.

(a) Define a predicate $M(x,y,z)$ = "number y belongs to the interval $x, \ldots, z$ excluding end points" using sentences of predicate logic so that your definition covers all natural numbers (represented in the way explained above).

(b) Provide a counter model, on the basis of which your definition does not entail

$$\forall x \forall y \forall z (M(x,y,z) \rightarrow M(z,y,x)).$$

Assignment 5 (10p)

(a) Derive for the program `if(x<y) then {z=y-x} else {z=x-y}` the *weakest precondition* strarting from the *postcondition* `(z>0)`. (4p)

(b) Consider the following program `Swap`:

```
z=y-x; while(!(z==0)) {x=x+1; y=y-1; z=z-1}.
```

Use weakest preconditions and a suitable invariant (6p) to establish

$\models_p$ `[(x==n) && (y==m)] Swap [(x==m) && (y==n)]`.

The name of the course, the course code, the date, your name, your student number, and your signature must appear on every sheet of your answers.