T-61.3015 DIGITAL SIGNAL PROCESSING AND FILTERING

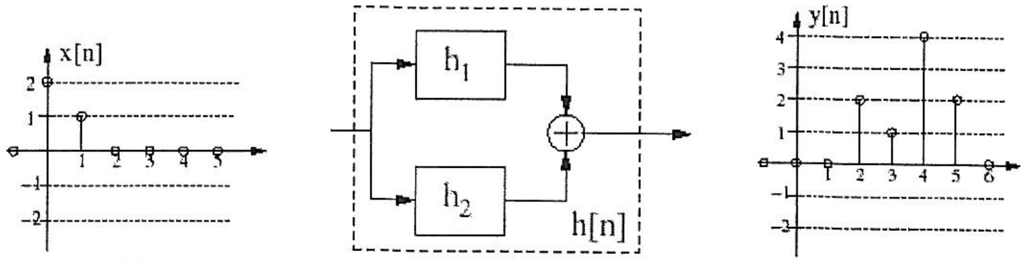
Examination / 11.1.2012 / OS

1. Are the following statements true or false? Correct answer: +1p, no answer: 0p, wrong answer: -1p. There are nine statements. The Maximum of the problem is six (6) points and minimum zero (0) points.
   (a) Multiplication in the time domain corresponds to convolution in the frequency domain
   (b) If f_s is the sampling frequency, frequencies in the interval $2f_s \ldots 5/2f_s$ will fold into the interval $0 \ldots f_s/2$
   (c) The sequence $x[n]=\cos^2\left(\frac{2\pi}{15}n\right)$ is periodic and the period is N=15
   (d) The order of the filter $H(z)=\frac{p_{10}+p_{11}z^{-1}}{d_{10}+d_{11}z^{-1}}+\frac{p_{20}+p_{21}z^{-1}+p_{22}z^{-2}}{d_{20}+d_{21}z^{-1}+d_{22}z^{-2}}$ is two
   (e) The system $h[n]=\delta[n]+\delta[n-1]+\delta[n-2]+\delta[n-3]$ has constant group delay and it is $\tau_g(\omega)=3$
   (f) Finite wordlength arithmetic may cause limit cycles and instability in FIR filter realizations
   (g) The entire frequency response of an analog filter is mapped into the usable frequency band ($0 \ldots f_s/2$) of an IIR digital filter using the impulse invariant method
   (h) All zeros of lowpass Butterworth and Chebyshev digital filter transfer functions designed by bilinear transformation lie on the unit circle at z = -1 in the z-plane
   (i) Gibbs phenomen refers to the oscillatory behavior of the magnitude responses of FIR filters. It can be reduced by selecting a suitable window function

(6p)

2. Consider an LTI system depicted in the figure below. It consists of two components, which are connected as shown in (b). The impulse response of the subsystem h_1 is $h_1[n]=\delta[n]-\delta[n-1]$. The impulse response $h_2[n]$ of the subsystem h_2 is unknown. If there is an input sequence $x[n]$ shown in (a) on the left the output sequence $y[n]$ will be as shown in (c) on the right, i.e.,

$$x[n]=2\delta[n]+\delta[n-1]$$
$$y[n]=2\delta[n-2]+\delta[n-3]+4\delta[n-4]+2\delta[n-5]$$



(a) (b) (c)

   (a) Compute the output of subsystem h_1: $y_1[n]=h_1[n]*x[n]$
   (b) Determine the two first values of the entire unit impulse response $h[n]$: $h[0]$ ja $h[1]$
   (c) Determine the impulse response $h_2[n]$ of the subsystem h_2
   (d) What is the output $y_m[n]$ when the input is $x_m[n]$=-$x[n$-1] ? Sketch the output sequence.

(6p)

TURN OVER !

3. Consider the finite impulse response (FIR) systems with the unit impulse responses given below

$$h_1[n] = \delta[n] + 2\delta[n-2] + \delta[n-4]$$

$$h_2[n] = \delta[n] - \delta[n-4]$$

(a) Determine the unit impulse response $h[n]$ and the transfer function $H(z)$ of the cascade connection of systems $h_1[n]$ and $h_2[n]$.

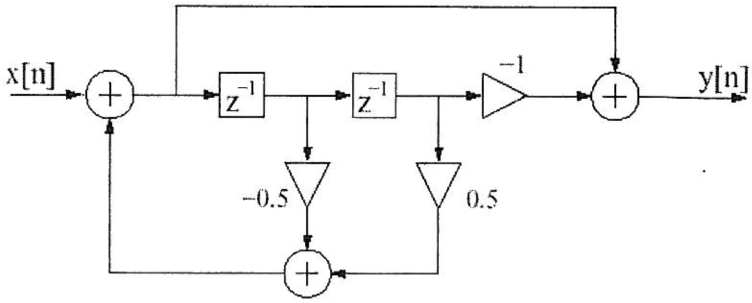
(b) Calculate the frequency response, i.e., the magnitude and phase responses, of the cascade system and sketch them graphically.

(c) Determine the step response of the cascade system. How does the step response behave with large values of n.

(d) How does the phase response of the parallel connection of $h_1[n]$ and $h_2[n]$ behave? Justify your answer!

(6 p)

4. Consider the digital filter structure shown in the figure below



(a) Determine the transfer function $H(z)$ in its simplest form.

(b) Is the realization canonic with respect to delays?

(c) Solve the poles and zeroes and sketch the pole-zero diagram.

(d) Sketch the magnitude of the frequency response $|H(e^{j\omega})|$.
Is the filter of lowpass, highpass, bandpass, or bandstop type?

(e) Scale the transfer function with constant K so that the maximum gain will be 1, i.e. $max\{|KH(e^{j\omega})|\}=1$

(f) Is the filter stable? Justify!

(6p)

5. Derive a recursive algorithm that generates the sequence n^3 (0,1,8,27,...). The algorithm is of the form

$$y[n] = \sum_{i=1}^{N} a_i y[n-i] + b$$

where a_i and b are constants. What are the necessary initial values? (6 p)

Hints: Investigate the consecutive terms of the sequence, e.g. using expressions $(n-1)^3$, n^3, $(n+1)^3$ and express them with y[n] and its shifted occurrences.

Another alternative is to consider the z-domain transfer function of the recursive generator. The generator does not have any input signal, but it can be "started" using a unit impulse input.

The z-transform of the sequence n^3 : $Z\{n^3\} = \dfrac{z(z^2+4z+1)}{(z-1)^4}$