Aalto University School of Science
Department of Information and Computer Science
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T-79.1001 Introduction to Theoretical Computer Science T (4 cr)
Exam Thursday 25 August 2011 9.00–12.00

Write on every answer sheet:
- Name, degree programme, student number
- The text: "T-79.1001 Introduction to Theoretical Computer Science T 25.8.2011"
- The total number of answer sheets submitted for grading

1. Show that the following languages are regular by describing each of them as a regular expression or as a finite state automaton:

   (a) $\{w \in \{a,b\}^* \mid w$ starts or ends with the substring $abba\}$ *5p.*

   (b) $\{w \in \{a,b\}^* \mid$ the number of bs in w is even$\}$ *5p.*

   (c) $\{w \in \{a,b,c\}^* \mid w$ does not contain the substring aa or the substring $ab\}$ *5p.*

2. Consider the language

$$L = \{b^k aab^n \mid n \geq k+1 \text{ and } k \geq 0\}.$$

   (a) Show that L is not regular. *7p.*

   (b) Design a context free grammar that produces L. *6p.*

   (c) Give parse trees for the strings aab and $baabbb$ in your grammar. *2p.*

3. Design a Turing machine that recognises the language

$$L = \{w \mid w \text{ contains equally many } a\text{s and } b\text{s}\}.$$

   If you wish, your machine may have multiple tapes. Present your machine as a state diagram and describe its method of operation verbally.

   Give the computation of your machine with the inputs ab and aca. *5p.*

   15p.

4. Let L_1 and L_2 be languages over an alphabet Σ.

   (a) Show that if the languages L_1 and L_2 are both regular, then the language $L = \{xy \mid x \in L_1 \text{ and } y \in L_2\}$ is also regular. *5p.*

   (b) Define the notions of a recursive ("decidable") and recursively enumerable ("semidecidable") language. Give an example of a language that is recursively enumerable, but not recursive. (You should provide a precise definition for the language, but need not prove any of its claimed properties.) *5p.*

   (c) Show that if the language L_1 is recursively enumerable and L_2 is recursive, then the language $L = L_1 \cap L_2$ is recursively enumerable. *5p.*

Total 60p.