

**Assignment 1** (Max. 10p)

- (a) Define/explain the following terms/mathematical concepts in detail (2p each):

a *minimal model*, an *antichain*, and a *full set*.

- (b) Which of the following claim are true and which false? Justify your answers precisely! (2p each)

Claim 1: For any normal programs  $P$  and  $Q$ : if  $P \equiv_s \emptyset$ , then  $P \cup Q \equiv Q$ .

Claim 2: For any *positive* normal programs  $P$  and  $Q$ : if  $\text{CM}(P) = \text{CM}(Q)$ , then  $P \equiv Q$ .

**Assignment 2** (Max. 10p) Consider a normal logic program  $P$  consisting of the following rules.

$$a \leftarrow a, b. \quad a \leftarrow \sim b. \quad b \leftarrow \sim c, d. \quad c \leftarrow \sim b, \sim e. \quad d \leftarrow \sim f. \quad e \leftarrow \sim d.$$

Determine the following (sets of) models for  $P$ :

- (a)  $\text{WFM}(P)$ , (3p)  
(b)  $\text{SM}(P)$  using the branch&bound algorithm and approximations, and (4p)  
(c)  $\text{SuppM}(P)$  using the completion  $\text{Comp}(P)$ . (3p)

**Assignment 3** (Max. 8p) Consider the following primitives for answer set programming:

- (a) Make  $b$  true if and only if all atoms except one amongst  $a_1, \dots, a_n$  are true.  
(b) Make an even number of atoms amongst  $a_1, \dots, a_n$  true.

How would you express these primitives using

- rule types available in the `smodels` system and
- basic/normal rules only?

**Assignments 4–5** are given on the reverse side of this sheet!!!

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The name of the course, the course code, the date, your name, your student number, and your signature must appear on every sheet of your answers.

**Assignment 4** (Max. 12p) Consider the problem of deciding whether two graphs  $G_1 = \langle V_1, E_1 \rangle$  and  $G_2 = \langle V_2, E_2 \rangle$  given as input are *isomorphic*, i.e., there is a bijection  $f : V_1 \rightarrow V_2$  such that for all nodes  $u, v \in V_1$ ,

$$\langle u, v \rangle \in E_1 \iff \langle f(u), f(v) \rangle \in E_2.$$

Suppose that the respective sets of edges  $E_1$  and  $E_2$  are represented using relations  $\text{Edge}_1(\cdot, \cdot)$  and  $\text{Edge}_2(\cdot, \cdot)$ .

Write a **normal** logic program  $P(G_1, G_2)$  which uses variables and input predicates  $\text{Edge}_1(\cdot, \cdot)$  and  $\text{Edge}_2(\cdot, \cdot)$  representing the graphs  $G_1$  and  $G_2$  such that  $\text{SM}(P(G_1, G_2)) \neq \emptyset \iff G_1$  and  $G_2$  are isomorphic.

Given this and the complexity results concerning normal logic programs, what can be stated about the computational time complexity of deciding graph isomorphism?

**Assignment 5** (Max. 10p) Consider the following normal program  $P$ :

$$\begin{array}{llll} a \leftarrow d. & a \leftarrow \sim e. & b \leftarrow e, \sim a. & \\ c \leftarrow b. & d \leftarrow a. & e \leftarrow c, \sim a. & e \leftarrow \sim d. \end{array}$$

- Form the *positive* dependency graph  $\text{DG}^+(P)$  of  $P$  and determine strongly connected components. (2p)
- Split the program in two *non-trivial* modules  $\mathbb{P}_1$  and  $\mathbb{P}_2$  such that  $\mathbb{P}_1 \sqcup \mathbb{P}_2$  is justifiably defined. Provide interface specifications for the modules  $\mathbb{P}_1$  and  $\mathbb{P}_2$ , and calculate the composition  $\mathbb{P}_1 \oplus \mathbb{P}_2$  (4p).
- Apply the *module theorem* in order to calculate  $\text{SM}(\mathbb{P}_1 \sqcup \mathbb{P}_2)$  using  $\text{SM}(\mathbb{P}_1)$  and  $\text{SM}(\mathbb{P}_2)$ . (4p)