

Mat-1.1620 Mathematics 2

1st midterm examination

22 February 2011 / Kibble

Fill in all the required information on each answer sheet. Calculators are not allowed. Examination time is 3 hours. Each problem is worth 3+3 points, adding up to 24 points.

1. Solve the following initial value problems:

(a) $\frac{dy}{dx} = -5x^4 y^2, \quad y(0) = 1$

(b) $y'' + 2y' + 5y = 0, \quad y(0) = 1, y'(0) = 5$

2. (a) Find the general solution to the Euler equation:

$$x^2 y'' + xy' - y = 0$$

(b) Find the general solution of the differential equation:

$$y'' + y' - 2y = 4x$$

3. (a) Find the area of the parallelogram whose vertices in the xy -plane are $(0,0)$, $(4,1)$, $(2,3)$ and $(6,4)$.

Hint: first obtain two adjacent edge vectors.

(b) Find an equation of the plane passing through the line $x + y = 2$, $y - z = 3$, and perpendicular to the plane $2x + 3y + 4z = 5$.

4. (a) Find the length of the helix

$$r(t) = (\cos t)\hat{i} + (\sin t)\hat{j} + (2t)\hat{k}$$

$$(0 \leq t \leq 2\pi).$$

(b) Let $z = \sin(x^2 y)$, where $x = st^2$ and $y = s^2 + \frac{1}{t}$. Use the Chain Rule to find

$$\frac{\partial z}{\partial s} \text{ and } \frac{\partial z}{\partial t}.$$