Aalto University, Institute of Mathematics Talponen/Alestalo/Havola

Mat-1.1120 C2

Spring 2011

1st. MIDTERM EXAM

Mon 21.2.2011 16:00-19:00

Simple calculators are allowed.

1. Let $f\colon \mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = x\sin(xy)$.

(a) Calculate $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$. (2p)

(b) Calculate $\frac{\partial^2 f}{\partial x \partial y}$ and $\frac{\partial^2 f}{\partial y \partial x}$. (2p)

(c) Calculate $\nabla f(1,1)$. (2p)

2. Let $f\colon \mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = xe^{xy}$.

(a) Find the tangent plane of the graph of f at $(1,1,e)$. (2p)

(b) Calculate the directional derivative $D_u f(1,1)$ where $u = (1,2)$. (2p)

(c) Approximate the value of $f(1.1\ ,\ 1.2)$ by using the previous calculation. (2p)

3. Calculate

(a) The Taylor series for the function $x\sin(x)$ about 0. (2p)

(b) The value of the integral

$$\int_0^1 x\sin(x)\,dx$$

with the precision of one decimal place by using Taylor polynomials. (4p)
Hint: Alternating series.

4. Find the closest point of the curve $y(x) = \frac{1}{x}$, $x > 0$, to the origin by using Lagrangian multiplier. (6p)