KAAVAKOKOELMA

Tätä kaavakokoelmaa saa käyttää kurssin tenteissä syksystä 2005 alkaen.

$$(P^{-1} + H'R^{-1}H)^{-1} = P - PH'(HPH' + R)^{-1}HP \tag{1.3.3-11}$$

where P is $n \times n$, H is $m \times n$, and R is $m \times m$.

An alternative version of the above is

$$(A + BCB')^{-1} = A^{-1} - A^{-1}B(B'A^{-1}B + C^{-1})^{-1}B'A^{-1} \tag{1.3.3-12}$$

$$f_x(x) \triangleq \frac{\partial f}{\partial x} = [\nabla_x f(x)']' = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix} \tag{1.3.5-3}$$

$$\phi_{xx}(x) \triangleq \frac{\partial^2 \phi(x)}{\partial x^2} = \nabla_x \nabla_x' \phi(x) = \begin{bmatrix} \frac{\partial^2 \phi}{\partial x_1 \partial x_1} & \cdots & \frac{\partial^2 \phi}{\partial x_1 \partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 \phi}{\partial x_n \partial x_1} & \cdots & \frac{\partial^2 \phi}{\partial x_n \partial x_n} \end{bmatrix} \tag{1.3.5-6}$$

Define the ***stacked vector***

$$y \triangleq \begin{bmatrix} x \\ z \end{bmatrix} \tag{3.2.1-1}$$

The notation

$$y \sim \mathcal{N}[\bar{y}, P_{yy}] \tag{3.2.1-2}$$

will indicate that the variable y is ***normally (Gaussian) distributed*** with mean

$$\bar{y} = \begin{bmatrix} \bar{x} \\ \bar{z} \end{bmatrix} \tag{3.2.1-3}$$

and covariance matrix (assumed nonsingular)

$$P_{yy} = \begin{bmatrix} P_{xx} & P_{xz} \\ P_{zx} & P_{zz} \end{bmatrix} \tag{3.2.1-4}$$

For x and z *jointly Gaussian*, as assumed in (3.2.1-2), the ***conditional mean*** is

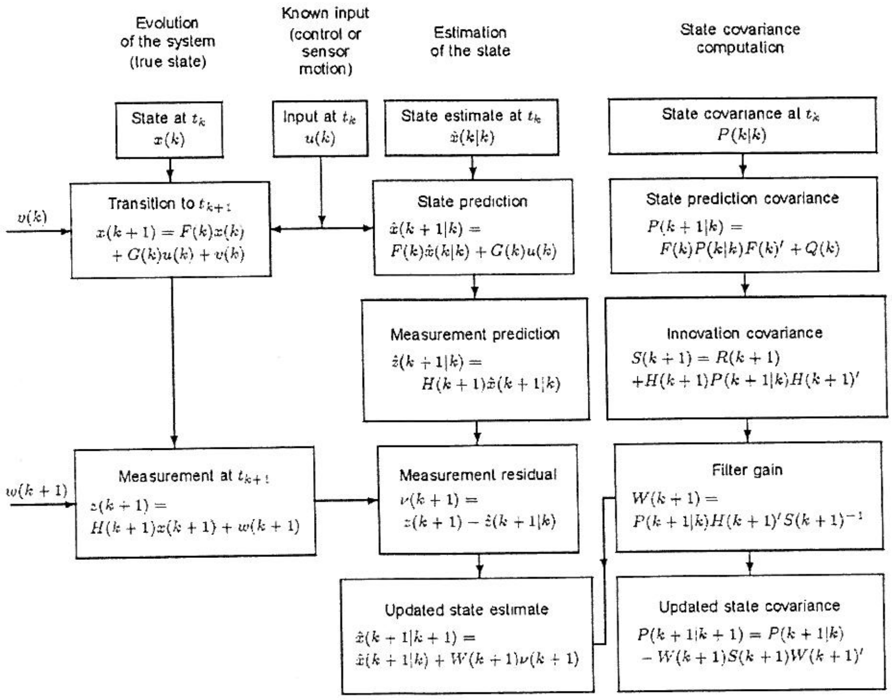
$$\boxed{\hat{x} \triangleq E[x|z] = \bar{x} + P_{xz}P_{zz}^{-1}(z - \bar{z})} \qquad (3.2.1\text{-}7)$$

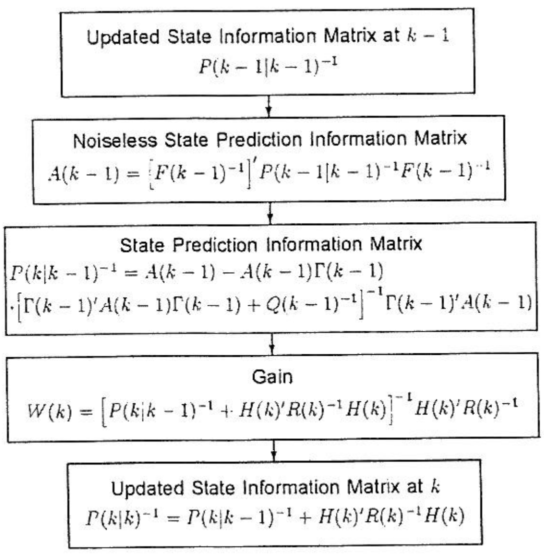
and the corresponding ***conditional covariance matrix*** is

$$\boxed{P_{xx|z} \triangleq E[(x - \hat{x})(x - \hat{x})'|z] = P_{xx} - P_{xz}P_{zz}^{-1}P_{zx}} \qquad (3.2.1\text{-}8)$$

$$\boxed{\hat{x}(k) = [H^{k'}(R^k)^{-1}H^k]^{-1}H^{k'}(R^k)^{-1}z^k} \qquad (3.4.1\text{-}9)$$

$$\boxed{P(k) = [H^{k'}(R^k)^{-1}H^k]^{-1}} \qquad (3.4.1\text{-}15)$$





$$\boxed{\hat{x}(k|N) = \hat{x}(k|k) + C(k)[\hat{x}(k+1|N) - \hat{x}(k+1|k)]} \tag{8.6.2-16}$$

where the ***smoother gain*** is

$$C(k) = P(k|k)F(k)'[F(k)P(k|k)F(k)' + Q(k)]^{-1} \tag{8.6.2-17}$$

or

$$\boxed{C(k) = P(k|k)F(k)'P(k+1|k)^{-1}} \tag{8.6.2-18}$$

$$\boxed{\begin{aligned} P(k|N) = P(k|k) + C(k)[P(k+1|N) \\ -P(k+1|k)]C(k)' \qquad k = N-1, \ldots, 0 \end{aligned}} \tag{8.6.2-29}$$