Quantum physics TFY-3.4323
Midterm exam
7 March, 2012

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Please write your answers either in English, Finnish, Swedish or German.

1. Explain briefly (you may use equations if you find it convenient, but it is not necessary):

   a) The difference in the description of bosons fermions in second quantization (2 points).
   b) Rotating wave approximation (2 points).
   c) Spontaneous emission and its connection to quantizing the electromagnetic field (2 points).

2. Consider the transition probability

$$P_{lk} = \frac{4|H_{kl}|^2}{\hbar^2(\omega_{kl}-\omega)^2}\sin^2\left[\frac{1}{2}(\omega_{kl}-\omega)t\right].$$

   Calculate the approximate result for P_{lk} and the rate $\frac{1}{t}P_{lk}$ for two limits: the long time limit $t \to \infty$, and the short time limit $t \ll (\omega_{kl}-\omega)^{-1}$. Compare the results to Fermi's golden rule. By the time-energy uncertainty relation, long (short) time of perturbation means small (large) uncertainty in energy. How is this reflected in the frequency dependence of the results?

3. A coherent state of a bosonic many-body system is given by

$$|\Psi\rangle = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle_{\text{Fock}},$$

   where $|n\rangle_{\text{Fock}}$ is the Fock state in which n particles have been created in some single-particle state i (let us denote that single-particle state $i=0$), i.e. $|n\rangle_{\text{Fock}} = \mathcal{N}\left(c_0^\dagger\right)^n|0\rangle$, where $|0\rangle$ is the vacuum state with zero particles.

   a) Find a proper normalization $\mathcal{N}$ for the Fock state $|n\rangle_{\text{Fock}}$ and calculate the norm of the coherent state $\langle\Psi|\Psi\rangle$.
   b) Find the number of particles in the coherent state $\langle\Psi|c_0^\dagger c_0|\Psi\rangle$.
   c) Find the variance of the number of particles in the coherent state $\langle\Psi|c_0^\dagger c_0 c_0^\dagger c_0|\Psi\rangle - \left(\langle\Psi|c_0^\dagger c_0|\Psi\rangle\right)^2$.

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4. The Hamiltonian in the field operator formulation is

$$\begin{aligned} H &= \int d^3x \left(\frac{\hbar^2}{2m} \nabla\psi^\dagger(\mathbf{x})\nabla\psi(\mathbf{x}) + U(\mathbf{x})\psi^\dagger(\mathbf{x})\psi(\mathbf{x}) \right) \\ &+ \frac{1}{2}\int d^3x \int d^3x'\, \psi^\dagger(\mathbf{x})\psi^\dagger(\mathbf{x}')V(\mathbf{x},\mathbf{x}')\psi(\mathbf{x}')\psi(\mathbf{x}), \end{aligned}$$

where U is an external potential, m the mass of the particle, and V the interaction potential. Starting from the Heisenberg equation of motion

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{x},t) = -[H,\psi(\mathbf{x},t)] = -e^{iHt/\hbar}[H,\psi(\mathbf{x},0)]e^{-iHt/\hbar}$$

derive the equation of motion for the field operator:

$$\begin{aligned} i\hbar\frac{\partial}{\partial t}\psi(\mathbf{x},t) &= \left(-\frac{\hbar^2\nabla^2}{2m} + U(\mathbf{x})\right)\psi(\mathbf{x},t) \\ &+ \int d^3x'\psi^\dagger(\mathbf{x}',t)V(\mathbf{x},\mathbf{x}')\psi(\mathbf{x}',t)\psi(\mathbf{x},t). \end{aligned}$$

How does the form of this equation relate to the name "second quantization"?