Quantum physics TFY-3.4323
Final examination
15 May 2009

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1. Consider a single particle in the ground state of a one-dimensional box of length L with infinite walls (i.e. a potential of the form $V(x) = 0$ when $0 \leq x \leq L$ and $V(x) = \infty$ otherwise). Describe the time evolution of the system when the length of the box is expanded from L to $2L$

   a) adiabatically.

   b) suddenly.
   (Here you probably end up with some tricky integrals containing products of trigonometric functions. This form is enough for your answer and you do not need to evaluate these integrals.)

   c) Discuss the validity of these two approximations if instead of an expansion the size of the box shrinks from L to $\frac{1}{2}L$.

2. A coherent state of a bosonic many-body system is given by

$$|\Psi\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle_{\mathrm{Fock}},$$

   where $|n\rangle_{\mathrm{Fock}}$ is the Fock state in which n particles have been created in some single-particle state i (lets denote that single-particle state $i = 0$), i.e. $|n\rangle_{\mathrm{Fock}} = \mathcal{N} \left(c_0^\dagger\right)^n |0\rangle$, where $|0\rangle$ is the vacuum state with zero particles.

   a) Find a proper normalization $\mathcal{N}$ for the Fock state $|n\rangle_{\mathrm{Fock}}$ and calculate the norm of the coherent state $\langle\Psi|\Psi\rangle$.

   b) Find the number of particles in the coherent state $\langle\Psi|c_0^\dagger c_0|\Psi\rangle$.

   c) Find the variance of the number of particles in the coherent state $\langle\Psi|c_0^\dagger c_0 c_0^\dagger c_0|\Psi\rangle - \left(\langle\Psi|c_0^\dagger c_0|\Psi\rangle\right)^2$.

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3. Assume a system with the Hamiltonian $H = Ec^\dagger c$, where $c^{(\dagger)}$ is the annihilation (creation) operator of a fermionic or a bosonic particle in some single-particle state $|\phi\rangle$ (and for the sake of simplicity we limit ourselves to a single state). Let us define a density matrix $\rho = \frac{1}{\mathcal{Z}} e^{-\beta H}$.

   a) Find the normalization coefficient $\mathcal{Z}$ and calculate the average occupation number $\langle c^\dagger c\rangle = \mathrm{Tr}\left\{\rho c^\dagger c\right\}$ when the particles (and hence the c operator) are fermions.

   b) Find the average occupation number $\langle c^\dagger c\rangle$ when the particles are bosons.

   c) Give an example of a density matrix of a physical state which can not be described by a single state vector (i.e. wavefunction). Show that this is indeed the case for your example.

4. a) Explain in a few sentences what a quantum bit (qubit) is. Give an example of an experimental realisation of a quantum bit.

   b) What is the no-cloning theorem? What are its implications for quantum computing?

   c) Prove the no-cloning theorem.

5. Explain in a few sentences:

   a) Dipole approximation,

   b) Avoided crossing,

   c) Quantum phase transition,

   d) Cooper pair.