

Discrete Mathematics
Mat-1.2991
Hakula/Seiskari

## T

2. Midterm Exam, 14.5.2011

- Calculators are permitted.
- Every problem carries equal weight.
- In every answer state clearly the mathematical principles and reasoning behind your solution. Brute force methods are not part of the course.

Exercise 1 Consider the triangle group $G_{\triangle}$ :

| $\boldsymbol{a} \cdot \boldsymbol{b}$ | $\boldsymbol{b}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $i$ | $r$ | $s$ | $x$ | $y$ | $z$ |  |
| $\boldsymbol{a}$ | $\boldsymbol{a}$ | $i$ | $r$ | $s$ | $x$ | $y$ | $z$ |
|  | $s$ | $s$ | $i$ | $r$ | $y$ | $z$ | $x$ |
|  | $x$ | $x$ | $z$ | $y$ | $i$ | $s$ | $y$ |
|  | $y$ | $y$ | $x$ | $z$ | $r$ | $i$ | $s$ |
|  | $z$ | $z$ | $y$ | $x$ | $s$ | $r$ | $i$ |

Describe explicitly the partition of $G_{\triangle}$ by the left cosets of the subgroup $H=\{i, x\}$. Check that the partition is not the same as that given by the right cosets of $H$.

ExErcise 2 Show that in any field of characteristic $\operatorname{char}(F)=p$

$$
(x+y)^{p}=x^{p}+y^{p} .
$$

Exercise 3 Construct a code $C$ in $V^{6}$ which will encode five messages and correct one error using any method of your choice. Is the code unique?

Exercise 4 A windmill has four two-parted wings at right angles to each other. All eight pieces of the windmill are identical. Assuming that the axle is fixed, that is, only planar moves are allowed, find the number of 2-colourings of the windmill.

