Aalto University School of Electrical Engineering
Department of Communications and Networking
S-38.3143 Queueing Theory, Fall 2011

Examination
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1. Consider a single-server queue. The system is empty at time 0. New customers arrive at times 1, 2, and 4. Their service times are 7, 3, and 3, respectively. For each of the three service disciplines given below, determine the departure times of all three customers:

   (a) FIFO, (b) PS, (c) LIFO-PR.

2. Let (T_n) and (τ_n) be a renewal sequence and the corresponding sum sequence, respectively, so that $\tau_0 = 0$ and $\tau_n = T_1 + \ldots + T_n$ for all $n = 1, 2, \ldots$. Assume that the T_n are exponentially distributed with mean $1/\lambda$. Define a continuous-time and continuous-state process $Y(t)$ with state space $[0, \infty)$ by
$$Y(t) := e^{-(t-\max\{\tau_n : \tau_n \le t\})}. \tag{1}$$
   By utilizing the theory of regenerative processes, determine the steady-state mean value $E[Y]$ (as a function of λ).

3. Consider an M/E_2/1-FIFO queue with $\rho < 1$. So we assume that the service times follow the Erlang distribution with two phases,
$$P\{S \le x\} = 1 - e^{-2\mu x}(1 + 2\mu x).$$
   Let $X(t)$ denote the queue length at time t. In addition, let $Z(t)$ denote the phase of the customer in service (if any). If the system is empty, then $Z(t) = 0$. The pair $(X(t), Z(t))$ is a Markov process.

   (a) Draw the state transition diagram of the Markov process $(X(t), Z(t))$.

   (b) By utilizing the Pollaczek-Khinchin mean value formula, determine the mean steady-state waiting time $E[W]$ (as a function of λ and μ).

4. Consider a tandem queueing system consisting of two service stations, each with a single server and an infinite number of waiting places. New customers arrive at station 1 according to a Poisson process with intensity 0.05 arrivals per second. The service times in station 1 are independent and exponentially distributed with mean 16 seconds, and customers are served according to the FIFO service discipline. After the service is completed in station 1, each customer continues to station 2. The mean (steady-state) time that each customer spends in station 2 is 40 seconds. Determine the mean (steady-state) number of customers in the whole tandem queueing system.

5. Consider an M/G/1 queue with $\rho < 1$. By applying the Gittins index approach, justify the optimality of the SPT discipline with respect to the mean delay among the non-preemptive service disciplines Π^{NPR}.