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Department of Information and Computer Science
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T-79.1001 Introduction to Theoretical Computer Science T (4 cr)
Exam Tuesday 20 December 2011 13.00–16.00

Write on every answer sheet:
- Name, degree programme, student number
- The text: "T-79.1001 Introduction to Theoretical Computer Science T 20.12.2011"
- The total number of answer sheets submitted for grading

1. Finite state automata ja regular expressions.

   (a) Show that the language $\{w \in \{a,b\}^* \mid w \text{ has an odd number of } as\}$ is regular by describing it as a regular expression. *5p.*

   (b) Show that the language $\{w \in \{a,b,c\}^* \mid w \text{ does not contain the substring } aa \text{ or the substring } ca\}$ is regular by describing it as a finite state automaton. *5p.*

   (c) Design the deterministic finite state automaton with the minimal number of states that accepts the language described by the regular expression $bc(abc \cup ac^*)^*$. *5p.*

2. Let $L_1 = \{a^i b^{2i} \mid i \geq 1\}$ and consider the language

$$L = L_1^* = \{x_1 \ldots x_n \mid n \geq 0 \text{ and } x_1, \ldots, x_n \in L_1\}.$$

   (a) Show that L is not regular. *6p.*

   (b) Design a context free grammar that produces L. *6p.*

   (c) Give parse trees for the strings abb and $aabbbbabb$ in your grammar. *2p.*

   (d) Design a pushdown automaton that decides the language L. *6p.*

3. Design a Turing machine that recognises the language

$$L = \{w \mid w \text{ contains equally many } as \text{ and } bs\}.$$

   If you wish, your machine may have multiple tapes. Present your machine as a state diagram and describe its method of operation verbally.

   Give the computation of your machine with the inputs ab and aca. *10p.*

4. (a) Define the notions of a recursive ("decidable") and recursively enumerable ("semidecidable") language.
   Give an example of a language that is recursively enumerable, but not recursive. (You should provide a precise definition for the language, but need not prove any of its claimed properties.)
   Is the language $L_{\text{prime}} = \{x \in \{0,1,\ldots,9\}^* \mid x \text{ is a prime number}\}$ recursive or recursively enumerable? Justify your answer briefly. *5p.*

   (b) Prove the following claim either correct or incorrect: If L_1 and L_2 are context-free languages, then $L_1 \cap L_2$ is also a context-free language. *5p.*

   (c) Define "Rice's theorem". Use it to prove that the following problem is undecidable: given an arbitrary Turing machine, is the language it accepts regular? That is, prove that $L_{\text{reg}} = \{c \in \{0,1\}^* \mid L(M_c) \text{ is regular}\}$ is not recursive, where M_c denotes the Turing machine encoded by the string c. *5p.*

Total 60p.