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EXAM
16.12.2011

S-72.2410 Information Theory

1. (6p.) Entropy.

   (a) (2p.) Determine $H(X|Y)$ and $H(X,Y)$ when it is known that $H(X) = 1.1$ bits, $H(Y) = 1.2$ bits, and $I(X;Y) = 0.9$ bits.

   (b) (2p.) A source X has an alphabet $\mathcal{X}$ of size 3. Can the source have entropy 2? Can it have entropy -1? Motivate.

   (c) (2p.) The joint distribution of two variables (X,Y) can be presented as a 2×2 table:

$X \backslash Y$	0	1
0	a	b
1	c	d

   Give one possible set of values for a, b, c, and d, which fulfills the requirements that (i) $p(X = 0) = 0.3$, and (ii) X and Y are independent. (There are many possible solutions, but one is enough.) Motivate.

2. (6p.) Concepts and terminology. Connect each entry in (i)–(vi) to *exactly one* related entry in (1)–(9). (So three entries of those in (1)–(9) will not be connected to anything.)

(i) error correction	(1) channel coding
(ii) multiple access	(2) lossy source coding
(iii) Markov chain	(3) comparing probability distributions
(iv) Gaussian channel	(4) many receivers, one sender
(v) Lempel-Ziv	(5) symmetric channel
(vi) BSC	(6) lossless source coding
	(7) continuous alphabet
	(8) many senders, one receiver
	(9) stochastic process

3. (6p.) Channel capacity. Motivate your answers in a concise way.

   (a) Determination of the capacity of a copper wire telephone line with signals band-limited to $W = 3300$ Hz and with a signal-to-noise ratio (SNR) of 33 dB gives a capacity of approximately 36000 bits per second. How is it then possible for (A)DSL (Internet) connections—over the same copper wires—to transmit millions of bits per second?

   (b) (2p.) Assume that you want to transmit data (like pictures) from Pluto at a rate that is 100 times higher than the rate of current space probes. What is the core issue to achieve this? Unlimited bandwidth is assumed.

(c) (2p.) Given three binary symmetric channels available with crossover probabilities $p = 0.30$, $p = 0.50$, and $p = 0.60$, respectively, which one would you choose to use? What if the three probabilities are $p = 0.20$, $p = 0.70$, and $p = 0.90$?

4. (6p.) Source coding.

(a) (4p.) A source X has an alphabet $\mathcal{X}$ of eleven symbols

$$\texttt{a,b,c,d,e,f,g,h,i,j,k},$$

all of which have equal probability, $1/11$. Find an optimal binary uniquely decodable code for this source. How much greater is the expected length of this code than the entropy of X?

(b) (2p.) Find a binary Shannon code for the source in 4(a) and determine its expected length.