## S-72.1110 Signals and systems

Exam 16.12.2011

Answer Question 1, of the Questions 2 – 7 the four best performed are taken into account.

Intermediate steps should be included in the answers, just writing down the final answer is not enough.

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1. Give short answers to the following tasks, use figures when needed.

a) Use probability distribution functions and expected value to define the conditions for two random variables to be
   i) statistically independent
   ii) uncorrelated

b) Describe what is Gibbs' phenomenon.

c) A 1024 point DFT is utilized to study a signal in 0…5 MHz band. What is the time and frequency domain sampling interval in this case?

d) The receiver sensitivity of an IEEE 802.15.4 standard radio is -98 dBm. Determine the power in Watts.

e) A signal contains 1 Hz and 4 Hz frequency components. The signal is sampled using sampling frequency 4 Hz. Determine what frequencies appear in the sampled signal.

f) How are the amplitude and delay distortion of linear system defined from the transfer function $H(f)$?

2.

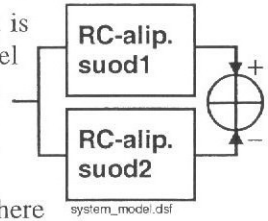
The raised-cosine filter is a filter frequently used for pulse-shaping in digital modulation due to its ability to minimize inter-symbol interference (ISI). The Fourier-transform of the simplest raised cosine filter is

$$H(f)=\frac{1}{2}\left(1+\cos(\pi fT)\right),\quad |f|\le\frac{1}{T}$$

Determine the impulse response of the filter $h(t)$.

3.

The output signal of the system shown in the right is the difference between the outputs of the two parallel RC low-pass filters. The impulse responses of the filters are $h_1(t)=\frac{1}{T}e^{-t/T}\,u(t)$ for RC-alip. suod1 and $h_2(t)=\frac{1}{2T}e^{-t/2T}\,u(t)$ for RC-alip. suod2 where




$$u(t)=\begin{cases}0 & t<0\\ 1 & t\ge 0\end{cases}$$ is the step function.

a) Determine the impulse response of the system

b) Determine the step response of the system by using (graphical) convolution.

4.

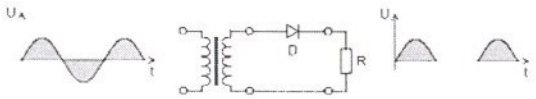
Let us assume that the 3 dB bandwidth of a Butterworth low-pass filter is 40 kHz. We require further that at 120 kHz the attenuation is to be at least 25 dB. What is the lowest order that a Butterworth filter meeting this specification can have?

The amplitude function of the filter is

$$A(f)=\frac{1}{\sqrt{\left(\frac{f}{W}\right)^{2n}+1}}$$

5.

A rectifier is an electrical device that converts alternating current (AC), which periodically reverses direction, to direct current (DC), which flows in only one direction. The process is known as rectification. The figure below illustrates half-wave rectification. The input signal is a sinusoid having voltage U =230 V and frequency is 50 Hz.



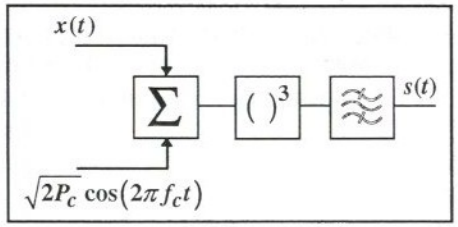

a) Determine Fourier-series representation of the output signal of the rectifier.

b) Determine the total harmonic distortion of the output signal.

6.

a) Show that the output of the 3rd order non-linearity in the adjacent system contains a DSB modulation term, and give the corresponding carrier frequency.

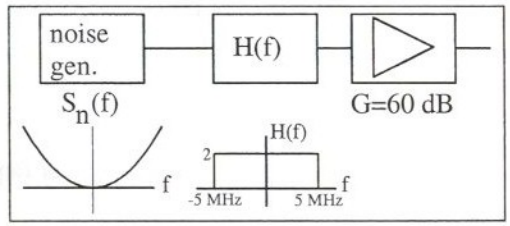
b) Dimension $f_c$ so that after the ideal band-pass filter one gets and undistorted DSB-signal, when the modulating signal bandwidth is $W_x$.




7.

a) Give the symbolic output **power** spectral density expression of the above system.

b) How is the average power calculated, when the power spectral density is known?




c) Calculate the output power (W) in the above system, when the noise generator produces parabolic noise with the power spectral density $S_n(f) = \frac{N_o}{2B^2} f^2$, where $\frac{N_o}{2} = 10^{-12}\, W/Hz$, and $B = 5$ MHz.