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- Calculators are permitted.
  - Every problem carries equal weight.
  - In every answer state clearly the mathematical principles and reasoning behind your solution. Brute force methods are not part of the course.

EXERCISE 1 Let us use the alphabet-number conversion:

$$\{A \rightarrow 00, B \rightarrow 01, C \rightarrow 02, \dots\}.$$

Note, that leading zeros can always be ignored in computations, e.g.,  $BB \rightarrow 0101 = 101$ .

Romeo and Julia have the RSA keys

$$(E_R, n_R) = (5, 19 \cdot 31) \quad \text{and} \quad (E_J, n_J) = (3, 11 \cdot 37),$$

respectively.

- a) Using Romeo's key, encode the message "WL".
- b) Write down the functions needed for the digital signature, when Romeo signs his message with 'R' so that only Julia can verify that the sender is indeed Romeo. Here the signature is considered a second message.

EXERCISE 2 Find all solutions to

$$\begin{aligned}x &\equiv 1 \pmod{3} \\x &\equiv 2 \pmod{5} \\x &\equiv 3 \pmod{2011}.\end{aligned}$$

EXERCISE 3 How many poker hands of five cards can be dealt from a standard deck of 52 cards? Also, how many ways are there to select 47 cards from a standard deck of 52 cards?

EXERCISE 4 Consider the expression  $(x + y + z)^{20}$ .

- a) How many terms are there in the expansion of the expression?
- b) Find the coefficient of the the term  $x^4 y^7 z^9$ .