Mat-2.3140 Linear Programming **Toppila/Kivistö**
Final exam **15.12.2011**

The exam consists of five problems that are graded on a scale 0-6. Each part of a problem is worth equally many points unless otherwise is stated.

Problem 1

Are the following statements right or wrong? Justify your answer.

(a) A basis is degenerate if and only if the corresponding basis matrix is singular.
(b) At optimum the reduced cost of a variable can deviate from zero only if the variable is zero.
(c) A problem in standard form is infeasible if and only if its dual is infeasible or unbounded.
(d) In practice all algorithms that are applicable for solving large integer optimization problems are polynomial time.

Problem 2

(a) Out of five alternatives one wishes to select as many as possible under the following constraints:
- Alternative 1 cannot be selected unless alternative 2 or 3 have been selected.
- At least 1 but at most 2 of alternatives 2-5 must be selected.
- Alternative 5 can be selected only if alternative 1 have not been selected, unless alternatives 3 and 4 have been selected.

Formulate the problem as a linear integer programming problem.

(b) Let the matrix $A \in \mathbb{R}^{n \times m}$, the vectors $b \in \mathbb{R}^n$ and $c \in \mathbb{R}^m, c \geq 0$ be given and the decision variable be $x \in \mathbb{R}^m$. Formulate as a (continuous) linear programming problem

$$\min \quad \sum_{j=1}^{m} c_j |x_j| \;\text{ s.t. } Ax \geq b \;.$$

Problem 3

Consider the problem

$$\begin{aligned} \min \quad & 13x_1 + 10x_2 + 6x_3 \\ \text{s.t.} \quad & \text{con1:}\;\; 5x_1 + x_2 + 3x_3 = 8 \\ & \text{con2:}\;\; 3x_1 + x_2 = 3 \\ & \qquad\quad x_1, x_2, x_3 \geq 0 \;. \end{aligned} \tag{1}$$

(a) Form the initial Simplex tableau for the basis consisting of x_2 and x_3.
(b) What are the elements in the Simplex tableau? Give the elements a geometric interpretation.
(c) Solve the problem with the Simplex-algorithm starting from the initial tableau.
(d) Formulate the dual of the problem and plot the path of the Simplex-algorithm to the dual space.

Problem 4

Let us continue with the linear program (1) in problem 3. In Table 1 is the sensitivity analysis report of this problem. Answer using the report (without solving the problem again) the following questions.

(a) How much can the unit cost of x_2 change without it entering the basis? What about x_3?

(b) Which is better: (i) reduce the right hand side (RHS) of con1 2 units if it costs 3 units or (ii) increase the RHS of con2 2 units if costs can reduced 6 units?

(c) To the problem is added a non-negative variable x_4 and the corresponding unit cost $c_4 = -2$ and constraint matrix column $A_4 = (7, -6)$. Does the optimal basis change?

(d) Assume that to the problem is added an inequality constraint that the current optimum does not satisfy. How can the optimal Simplex tableau of the original problem be utilized in starting the dual simplex algorithm?

Problem 5

Consider the uncapasitated network flow problem in Figure 1, where next to each arc is the arc cost and with the double arrow is denoted the demand or supply of the nodes.

(a) Formulate the problem as a standard form linear program.

(b) Solve the problem with the network simplex algorithm starting with the tree indicated with dashed arcs in the figure.

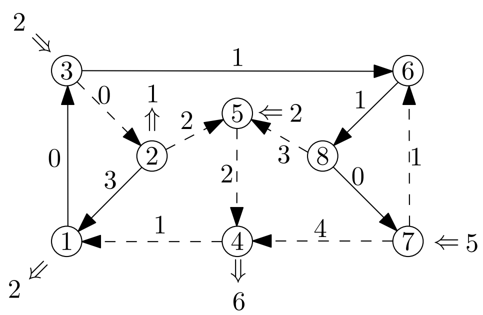

Figure 1: Network flow problem.

Table 1: Sensitivity analysis report.

No.	Row name	St	Activity	Slack / Marginal	Lower bound / Upper bound	Activity range	Obj coef range	Obj value at break point	Limiting variable
1	con1	NS	8.00000	.	8.00000	5.00000	-Inf	13.00000	x[3]
				2.00000	8.00000	+Inf	+Inf	+Inf	
2	con2	NS	3.00000	.	3.00000	.	-Inf	16.00000	x[1]
				1.00000	3.00000	4.80000	+Inf	20.80000	x[3]
3	objective	BS	19.00000	-19.00000	-Inf	40.00000	-1.00000	.	x[2]
				.	+Inf	19.00000	+Inf	+Inf	

No.	Column name	St	Activity	Obj coef / Marginal	Lower bound / Upper bound	Activity range	Obj coef range	Obj value at break point	Limiting variable
1	x[3]	BS	1.00000	6.00000	.	1.66667	-25.50000	-12.50000	x[2]
				.	+Inf	1.00000	+Inf	+Inf	
2	x[2]	NL	.	10.00000	.	-4.50000	3.00000	-12.50000	x[3]
				7.00000	+Inf	3.00000	+Inf	40.00000	x[1]
3	x[1]	BS	1.00000	13.00000	.	1.00000	-Inf	-Inf	
				.	+Inf	-Inf	34.00000	40.00000	x[2]