

Write in each answer paper your name, department, student number, the course name and code, and the date. Number each paper you submit and denote the total no. of pages. 5 problems, 30 points total. Papers in English only. The BETA mathematical tables can be utilized – you can borrow a copy from the exam supervisor if you do not have your own. A basic calculator can be used (no memory, no graphics).

The homework bonus will be valid for possible future exams too.

1. (1p each) Define and describe briefly (2..3 lines of text) the following concepts:

- a) Echo cancellation
- b) Stochastic Gradient (SG) algorithm
- c) Roll-off factor
- d) Matched filter
- e) Water-pouring theorem
- f) Nyquist criterion

2. Linear equalizers (6p):

Describe and compare the ZF, the MMSE, the DFE, the Viterbi equalizers. Discuss their advantages and disadvantages for different types of transmission channels, background noise levels, implementation complexities etc.

3. Matched filtering (6p):

Consider a discrete-time receive filter $h_R(k)$ and its frequency response $H_R(e^{j\omega k})$. Assume a simple discrete-time transmit filter:

$$h_T(k) = 3\delta(k) - \delta(k+2) + 2\delta(k-1) \quad (1)$$

- a) (3p) Find the matched-filter receive filter $h_R(k)$ and draw the impulse responses $h_T(k)$ and $h_R(k)$, both the ideal *noncausal* and *causal* versions.
- b) (3p) Determine the pulse waveform $g(k)$ at the output of the receive filter either via convolution:

$$g(k) = h_R(k) * h_T(k) = \sum_{l=-\infty}^{\infty} h_R(l)h_T(k-l) \quad (2)$$

or in the frequency domain if you prefer. Plot $g(k)$.

4. Nyquist and Matched filtering criteria (6p bonus):

Let us derive pulse waveforms which meet the Nyquist criterion:

- a) (2p) Assume an ideal baseband communication system of data rate $1/T$ in an AWGN channel which *needs no excess bandwidth*. Define the *transmit pulse spectrum* and derive the *continuous-time transmit pulse waveform* via inverse Fourier transform. No filtering assumed in the receiver, just symbol-rate sampling.
- b) (2p) The same as a) except now we assume excess bandwidth α ($0 < \alpha < 1$) and that the spectrum is *piecewise constant*.

- c) 3p) The same as b) except now we want to design *transmit* and *receive* filters that form a matched-filter pair and whose convolution meets the Nyquist criterion. Assume again that the spectrum of the convolution is piecewise constant.

5. Channel capacity (8p) [Bonus]:

Consider the transmission of signal $x(t)$ over a linear channel with associated impulse response $c(t)$ and frequency response $C(f)$. The output waveform of the channel is then $r(t) = c(t)*x(t)$, where $*$ denotes convolution. The output of the channel is thereafter corrupted by colored noise $n(t)$ with power spectral density (PSD) $S_n(f)$.

- (4p) Solve for the optimum transmit power spectrum $S_x(f)$ that maximizes the channel capacity when the total transmit power P_x is limited to 18 W (see Equation (5) on Page 3). To help you solve the problem, Figure 1 below provides you with the PSD of the noise $S_n(f)$ and the magnitude squared of the channel transfer function. Assume that $S_n(f) = \infty$, for $|f| > 4$ Hz.
- (2p) Determine the channel capacity obtained from the optimized transmit power spectrum in a).
- (2p) Provide the minimum transmit power that justifies the transmission in the whole frequency band $|f| < 4$ Hz.

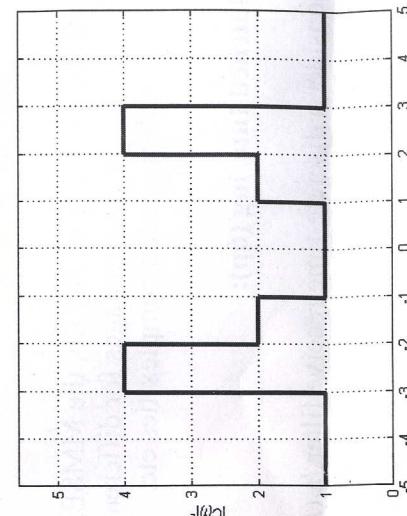
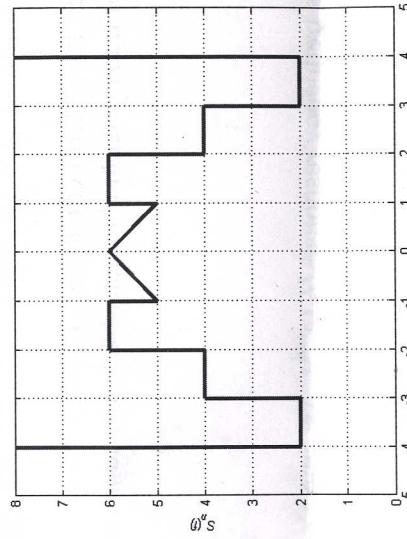


Figure 1: PSD of the noise $S_n(f)$ in W/Hz and channel gain $|C(f)|^2$ versus frequency f in Hz.

Hint: The optimal power spectrum is obtained with the water-pouring theorem as:

$$S_{x,opt}(f) = L - S_n(f) / |C(f)|^2 \quad (3)$$

whenever resulting $S_{x,opt}(f)$ is positive (zero otherwise) and the water-filling L is determined so that the total transmit power is limited, i.e.,

$$P_x = \int_{-\infty}^{\infty} S_{x,opt}(f) df \quad (4)$$

The optimal capacity is then obtained by (double-sided) integration:

$$C_{CH} = \frac{1}{2} \int_{-\infty}^{\infty} \log_2 \left(1 + \frac{S_{x,opt}(f) |C(f)|^2}{S_n(f)} \right) df \quad (5)$$