Mat-1.3651 Matrix Computations Exam 19.5. 2012.

Write down, on each paper, your name and other necessary information.

1) **(6 points.)** Suppose an invertible $A \in \mathbb{R}^{n\times n}$ has an LU factorization and L and U are known. Give an algorithm which can compute the (i,j)-entry of A^{-1} in approximately $(n-j)^2 + (n-i)^2$ flops.

2) **(6 points.)** Suppose $L \in \mathbb{R}^{m\times n}$ with $m \geq n$ is lower triangular. Show how Householder matrices $H_1, \ldots, H_n$ can be used to determine a lower triangular $L_1 \in \mathbb{R}^{n\times n}$ so that

$$H_n \cdots H_1 L = \begin{bmatrix} L_1 \\ 0 \end{bmatrix}.$$

Hint: The second step in the 6-by-3 case involves finding H_2 such that

$$H_2 \begin{bmatrix} x & 0 & 0 \\ x & x & 0 \\ x & x & x \\ x & x & 0 \\ x & x & 0 \\ x & x & 0 \end{bmatrix} = \begin{bmatrix} x & 0 & 0 \\ x & x & 0 \\ x & x & x \\ x & 0 & 0 \\ x & 0 & 0 \\ x & 0 & 0 \end{bmatrix}.$$

3) **(6 points.)** Let $A \in \mathbb{R}^{n\times n}$ be diagonalizable. Suppose A has one eigenvalue λ_1 which is larger in modulus than other eigenvalues of A. Explain how the power method can be used to approximate λ_1. Give also reasons why the power method works.

4) **(6 points.)** Let

$$A = \begin{bmatrix} I & Y_2 & & & \\ & I & Y_3 & & \\ & & \ddots & \ddots & \\ & & & I & Y_k \\ & & & & I \end{bmatrix}.$$

How many steps at most GMRES needs to give the exact solution to $Ax = b$, regardless of b. (Hint: observe that a certain power of $A - I$ equals zero.)