path of length 3

o—o—o—o ?

almost every G has property Q if $Pr(G \in \Omega_n \text{ has } Q) \xrightarrow[n\to\infty]{} 1$

At least one path of length 3? 4 vertices, two fixed?

What about the complementary Pr? Pr(exist vertices u, v s.t. u≠v and there is no path of length 3 joining them)?

Aalto University
Department of Information and Computer Science
Pekka Orponen (tel. 25246)

T-79.5206 Combinatorial Models and Stochastic Algorithms (5 cr)
Exam Tue 22 May 2012, 9–12 a.m.

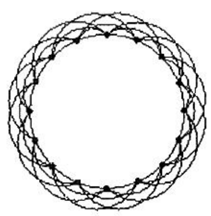
Permitted material at exam: lecture notes, any personal handwritten notes, tutorial problems and their solutions; calculator.

Write down on each answer sheet:
- Your name, study programme, and student id
- The text: "T-79.5206 Combinatorial Models and Stochastic Algorithms 22.5.2012"
- The total number of answer sheets you are submitting for grading

1. Let p, $0 < p < 1$, be a constant. Prove that in almost every ER random graph $G \in \mathcal{G}(n,p)$, any two vertices u, v, $u \neq v$, are connected by at least one path of length exactly 3. *12p.*

2. Compute the clustering coefficient $C(T)$ and characteristic path length $L(T)$ of an $n \times n$ torus graph T, i.e. an $n \times n$ square lattice with periodic boundary conditions: top/bottom and left/right boundaries merged so that the graph is regular of degree 4. *12p.*

3. Consider a *circulant graph* C_{nk}, i.e. a ring of n vertices where each vertex is connected to its k nearest neighbours along each direction in the ring. Thus, the graph $C_{16,2}$ looks like this:



Assume n is even and $k < \frac{n}{2}$. Recall that a *simple random walk* on a graph moves in each step from a vertex to one of its neighbours selected uniformly at random. Show that for any $k \geq 2$ the Markov chain corresponding to a simple random walk on C_{nk} is regular, and determine its stationary distribution. Calculate some upper bound on the mixing time of the chain. *12p.*

4. An *clique* in a graph $G = (V, E)$ is a set of vertices $V' \subseteq V$ every two of which are connected by an edge. (I.e. for any $u \neq v \in V'$, $\{u, v\} \in E$.) Design a simulated annealing approach to finding a maximally large clique in a given input graph G. Describe particularly clearly: (a) what are the candidate solutions considered by your method and what is their neighbourhood relation, (b) what is the objective function associated to the candidate solutions, (c) by what rule does one move from a given candidate solution to another one in its neighbourhood, and (d) how does one choose the initial candidate solution for the computation. Explain why the neighbourhood graph you have defined is connected, and present a cooling schedule that would guarantee that the algorithm eventually finds (with probability 1) a clique of maximum possible size. (In determining the cooling schedule, you may ignore the requirement that the neighbourhood graph be regular.) *14p.*

Total 50p.