T-61.3040 Statistical Signal Modeling
Exam 26.05.2012

In the exam, you are allowed to have a calculator (non-programmable or memory emptied) and basic mathematical tables (no tables containing material directly associated with the course). For example, the book "Beta Mathematics Handbook for Science and Engineering" by Rade and Westergreen contains material too directly associated with the course, so you are NOT allowed this book at the exam.

The results of the exam will be announced through the Noppa system. You can answer in English (preferred) or Finnish.

1. (max 10p) Explain the following topics briefly but covering the most important properties:

a) Wide-Sense Stationarity (WSS) (2p)

b) ARCH model (2p)

c) Wiener filter (3p)

d) The Minimum Variance Method (3p)

2. (max 10p) Given the following values of a real-valued process $x(n)$: $x(0) = 1$, $x(1) = 2$, $x(2) = 2$:

a) Estimate an autocorrelation matrix of size 3×3 so that the result is positive semi-definite. Show that this result is positive semi-definite. (3p)

b) Model the process $x(n)$ as an AR(2) process. (3p)

c) What are the variance $\mathrm{Var}(x(n))$ and conditional variance $\mathrm{Var}(x(3)|x(2), x(1), x(0))$ of the modeled process? (4p)

3. (max 10p) Multiple choices questions. The following five questions have different proposed answers. Only one of them is correct. You have to give your answer along with your confidence ("High" or "Low") for each answer. Grading for each of these questions is then:

- +2 if the answer is correct and confidence High
- +1 if the answer is correct and confidence Low
- 0 if the answer is missing
- −1 if the answer is wrong and confidence Low
- −3 if the answer is wrong and jonfidence High

Write on your answer sheet the correct answer A, B, C, D,...) along with the confidence you have (High or Low) for that question. For example, "A, Low" is a proper way of answering a question. No need to justify your answers. Total score for this question is between 0 and 10 (you cannot get a negative score).

1) The Pisarenko method is used to estimate the power spectrum of a WSS process $x(n)$. The pseudo-spectrum is calculated and:

   A) The pseudo-spectrum is a very good estimation of the power spectrum

   B) The pseudo-spectrum is infinite for the main frequencies of the process $x(n)$

   C) In the pseudo-spectrum, some peaks appear for the main frequency of the process $x(n)$

D) The pseudo-spctrum is equal to zero for the main frequencies of the proccss $x(n)$

E) In the pseudo-spectrum, some peaks appear for the main frequencies of the process $x(n)$

F) None of the previous answers is correct

2) Which of the following methods has the best resolution (you can assume than N is large and that the following methods parameters L, K, M have values around $\sqrt{N}$)?

A) Bartlett's method

B) Blackmann-Tukey's method

C) Welch's method

D) Periodogram method

3) The autocorrelation sequence $r_y(k)$ of the output $y(n)$ of a stable LSI filter with unit sample response $h(n)$ filtering a WSS process with autocorrelation $r_x(k)$ can be calculated as:

A) $r_y(k) = r_x(k) * h(k)$

B) $r_y(k) = r_x^*(-k) * h(k) * h^*(-k)$

C) $r_y(k) = r_x(k) * h(k) * h^*(-k)$

D) $r_y(k) = r_x(k) * r_{xy}(k)$

E) None of the previous answers is correct.

4) For jointly WSS processes, the LMS algorithm converges in the mean if the independence assumption is satisfied and the step size μ satisfies (with $\lambda_{\max}, \ldots, \lambda_{\min}$ the ordered eigenvalues of the autocorrelation matrix $\mathbf{R}_x$):

A) $\mu = \sum_{k=0}^{p} \lambda_k$

B) $0 < \mu < \frac{2}{\lambda_{\min}}$

C) $\mu < 0$

D) $0 < \mu < \frac{2}{\lambda_{\max}}$

E) Minimum of the power spectrum $< \mu <$ Maximum of the power spectrum

F) None of the previous answers is correct.

5) The estimate of the autocorrelation matrix using the covariance method is:

A) Unbiased and Toeplitz

B) Biased and Toeplitz

C) Biased and not Toeplitz

D) Unbiased and not Toeplitz

E) None of the previous answers is correct.

4. (max 10p) The process $x(n) = A\exp(jn\omega) + v(n)$ consists of one complex sinusoid in white noise. If the correlation matrix is

$$\mathbf{R}_x = \begin{bmatrix} 3 & 2(1-j) \\ 2(1+j) & 3 \end{bmatrix}$$

then:

a) What is the variance of the noise? (3p)

b) What is the power $P = |A|^2$? (3p)

c) What is the frequency ω of the signal? (4p)